

# An investigation on a prospective teacher's professional noticing skills in the process of narrative construction and discussion

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## Abstract

This qualitative study aims to investigate the ability of a prospective middle school mathematics teacher to notice student thinking through narratives and online discussions. Data were collected from the prospective teacher as part of the Teaching Practice course. The data for the study comprised three narratives written by the prospective teacher, along with the corresponding discussions held in online forums. Content analysis was employed to analyze the data. The findings revealed that the prospective teacher demonstrated a robust level of noticing student thinking in the attending component, a combination of robust and partial levels in the interpreting component, and a partial level in the responding component. These results are consistent with previous studies in the literature that highlight teachers' challenges in responding to student thinking. Conversely, it was noted that the prospective teacher exhibited some improvement in the responding component of noticing skills during online discussions. In a collaborative environment where ideas were exchanged, there were instances indicating that the prospective teacher enhanced her responding skills to a higher level. The findings were discussed in the light of the related literature, and suggestions were presented.

## 1 Introduction

Teacher noticing has been studied in the field of education for decades and can be considered as a key component considering that teachers' noticing skills have an impact on student learning (van Es & Sherin, 2021). Van Es and Sherin (2021) point out that teachers' noticing skills are important in terms of focusing on students' ideas and interacting with students based on these ideas and that it is a necessity for lessons to be built around student thinking. In other words, teachers' professional noticing skills are a requirement of lessons that have an impact on student learning and are centered on student thinking (van Es & Sherin, 2021). This skill is necessary for teachers to focus on student ideas in their lessons and to communicate effectively with students

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(van Es & Sherin, 2021). It is thought that this skill plays an important role in the instant and practical decisions made by teachers during teaching (Mason, 2002). At this point, it becomes possible for teachers to make more accurate instructional decisions and provide more effective teaching for student needs by improving their professional noticing skills (El Mouhayar, 2024).

The concept of learning to notice examines how teachers notice what and how they analyze what they notice in the classroom (van Es & Sherin, 2008). Van Es and Sherin (2002) identified two dimensions in their original theoretical framework of noticing: attending, which involves recognizing significant situations within a classroom environment, and interpreting, which entails analyzing those situations based on prior knowledge and experience while relating them to broader educational principles. Subsequently, the researchers expanded their framework by introducing a third dimension, shaping, in addition to the original two (van Es & Sherin, 2021). In this extended framework, the attending dimension allows teachers to identify critical moments in the teaching process while disregarding less important ones; the interpreting dimension enables teachers to examine these noticed points from a questioning perspective; and the shaping dimension encourages teachers to engage with their environment to gather more detailed information about the observed situations, thereby supporting their understanding. By incorporating this third dimension, the researchers shift the focus from merely participation and interpretation to the dynamic interaction between teachers and students, with the aim of acquiring further information that enhances noticing and facilitates the identification of new participatory and interpretive situations informed by this additional data. The researchers state that they started from the idea of decision-making by Blömeke (2015) and Jacobs et al. (2010) while creating this third dimension, but the shaping dimension differs from 'decision making' with the feature of acquiring additional knowledge through interaction instead of focusing on the next instructional action in order to improve student thinking.

Jacobs et al. (2010) are often recognized for their work on teachers' ability to notice student thinking. Their theoretical framework emphasizes a focus on observing students' cognitive processes. Specifically, Jacobs et al. (2010) introduced the concept of 'professional noticing of children's mathematical thinking' (p. 169), examining how effectively and to what extent teachers can identify students' mathematical thought patterns. To elaborate on this concept, the ability to notice student thinking consists of the components of attending to (being able to identify the student's solution strategy), interpreting (being able to interpret student thinking), and responding (being able to decide how to respond to student thinking). Accordingly, in the attending to component, beyond being able to notice remarkable situations in a classroom in general terms, being aware of student thinking and identifying and revealing student thinking; in the interpreting component, beyond being able to interpret the interactions in the classroom and the situations noticed in the light of learning and teaching principles in general terms, being able to interpret the student thinking revealed; in the responding component, beyond being able to respond to the situations noticed, it is about deciding how to take a step towards the student thinking specifically noticed. Here, the teacher decides on the next instructional move in order to take the student's thinking, which he/she has noticed and commented on the causes and possible consequences, one step further and develop it.

These skills are closely interconnected. A teacher's ability to accurately identify student thinking is crucial for effectively interpreting it. By providing detailed identification and interpretation, teachers are better equipped to make informed instructional decisions that enhance students' understanding. Jacobs et al. (2010) emphasize that identifying students' specific mathematical

strategies aids in interpretation and response during the noticing process. These findings are further supported by the research of Schack et al. (2013).

It is important for teachers to be aware of their students' thinking, to create learning opportunities for this, to listen to student ideas, and to teach based on student thinking (Ivars et al., 2020; Monson et al., 2020). At this point, as stated in the literature, providing prospective teachers with environments where they can use their skills of attending to, interpreting, and responding to student thinking contributes to the development of their professional noticing skills (Schack et al., 2013). It is important for teachers/prospective teachers to predict what they may think in the next step after identifying and interpreting student thinking, to decide how to respond to student thinking, and to ask the right questions to ensure student reasoning (Ivars et al., 2020), as Bergman et al. (2023) emphasized, it is especially necessary to develop the ability to respond to student thinking in teachers.

Jacobs et al. (2010) state that although experienced teachers' noticing skills are at a higher level, teachers, in general, have difficulty noticing. Teachers may have difficulties, especially in responding to student thinking (Franke et al., 2009). Instead of responding to student thinking by asking effective questions and supporting students' reasoning, teachers can ask overly directive questions and respond to student thinking with a procedural approach (Son, 2013). Especially when it is considered that prospective teachers do not have much instructional experience and may need more support in this sense (van Es & Sherin, 2008), it is essential to create developmental environments that can support them within teacher training programs.

In addition to providing opportunities for prospective teachers to develop their skills on what and how to notice in the development environments to be provided to prospective teachers (Stockero et al., 2017), efforts to increase their awareness of student thinking, in particular, will be beneficial (Jacobs et al., 2010). As stated by Schack et al. (2013), it is important to create appropriate learning environments in order to develop prospective teachers' noticing skills toward student thinking. It should be taken into consideration that noticing skill is not an innate skill in in-service/prospective teachers (Jacobs et al., 2010; van Es & Sherin, 2008), and there is a need for environments that will serve the development of this skill. Therefore, for the development of this skill, which is considered to be important, it is necessary to provide in-service and prospective teachers with instructional experience opportunities where they can develop these skills.

In the mathematics education literature, there are various studies on teachers' noticing skills. It is seen that these studies focus on the teacher training process, teacher education and/or lesson planning and evaluation process, and in addition to examining teacher knowledge and noticing skills on specific mathematics topics, there are also studies comparing the noticing skills of in-service and prospective teachers (i.e. Bas-Ader et al., 2021; Bonaiuti et al., 2020; Choy et al., 2017; Fernandez et al., 2020; Sanchez-Matamoros et al., 2015; Sherin et al., 2011). As stated by Amador et al. (2021), the theoretical frameworks of van Es (2011) and Jacobs et al. (2010) are mainly used in the studies conducted on prospective teachers' noticing skills. In learning environments where the professional development of in-service and/or prospective teachers is targeted, the development of noticing skills is aimed through group discussions (Sherin & Han, 2004), analyses of classroom videos (van Es & Sherin, 2002), analyses on examples of student thinking (Walkoe, 2015), and analyses of student thinking through narratives (Fernandez et al., 2020).

Narratives, whose use in the field of education dates back to the 1980s (Pulvermacher & Lefstein, 2016), have been recognized as one of the effective tools used in mathematics teacher education

since the mid-1990s, which can be used to reveal teachers' perspectives and develop their reflective thinking (Chapman, 2008). Chapman (2008) considers narratives as written or oral stories prepared by teachers themselves and defines them as tools through which teachers reveal their own perspectives on the teaching and learning process.

Narratives are considered a potentially effective approach to enable prospective teachers to reflect on practice (Pulvermacher & Lefstein, 2016). Chapman (2008) states the following about narratives: "They can facilitate interpretation and understanding of our experiences and offer a way of recovering, articulating, and understanding the meanings and intentions embodied in our behaviors." (p. 17). Ivars and Fernandez (2018) also state that narratives make it possible to understand teachers' teaching processes. Through the use of narratives, teachers have the chance to express their understanding of mathematics teaching (Chapman, 2008). Narratives are used in the professional development of prospective teachers as well as in-service teachers. Pulvermacher and Lefstein (2016) state that narratives, the most accessible form of representation of classroom practice, are often used in prospective teacher education methods courses. The narrative writing process can be seen as a learning opportunity for prospective teachers (Ponte et al., 2009). In particular, narratives can be beneficial in terms of making the teaching process visible and strengthening the connection between theory and practice by eliminating the disconnect between the process of learning to teach and teaching in real classrooms (Pulvermacher & Lefstein, 2016). In the process of writing narratives, prospective teachers can have the opportunity to capture important details about teaching by focusing on the mathematics content as well as the interaction in a classroom (Ivars & Fernandez, 2018). With the use of narratives in the context of professional noticing skills, it may be possible to examine the noticing skills of prospective teachers and to reveal how these skills differ. For example, El Mouhayar (2024) examined the differences in the ability of eight prospective teachers to notice students' mathematical thinking through analyses within the scope of the teaching practice course and found that prospective teachers exhibited different levels of noticing skills where the majority of them could not exceed the average noticing level. Moreover, considering that noticing should be directed towards student learning rather than easily observable points in order to talk about the development of professional noticing skills (Sherin & van Es, 2005), it can be thought that prospective teachers can be enabled to focus on student thinking through narratives. Ivars and Fernandez's (2018) study, which examined the noticing skills of 22 prospective teachers through narratives within the scope of attending to, interpreting, and responding components of noticing skill, points out that the process of writing narratives on teaching practice experiences and receiving feedback from the university tutor contributed to prospective teachers' ability to notice student thinking.

In another study, Fernandez et al. (2020) addressed teachers' noticing skills through narratives and examined the development of five prospective mathematics teachers' noticing skills in an online teacher education program. Within the scope of the study, the narratives of school observations written by prospective teachers during the teaching practice process were discussed online, and the contribution of the feedback provided by the instructor leading the course as well as other prospective teachers in the environment to the noticing skills of prospective teachers was examined. The narratives written by the prospective teachers included the interpretation of the mathematical situations that occurred during the teaching at the school where they did their internship by considering the possible reasons and discussing what should be done in the next instructional step. In this respect, the narratives cover the dimensions of the noticing skill. Fernandez et al. (2020) examined the changes and developments in prospective teachers'

narratives and tried to reveal the relationship between the feedback provided in the online environment and these changes. The findings of the study indicated that the online discussion environment benefited the development of prospective teachers' noticing skills. As Fernandez et al. (2020) stated, prospective teachers take the role of narrators of their own stories through narratives, and through narrative writing, they have the opportunity to construct their skills of noticing student thinking during their teaching practice experiences. In this sense, narratives can be useful in revealing the interaction of prospective teachers with students while students are engaged in mathematics (Fernandez et al., 2020).

Ivars and Fernandez (2018) emphasize that the development of prospective teachers' noticing skills throughout the teacher education program is important, but it is not an easy task. At this point, the teaching practice process provides suitable environments where prospective teachers' noticing skills can be developed (Fernandez et al., 2020). On the other hand, as Fernandez et al. (2020) underlined, there are not enough studies addressing prospective teachers' ability to notice students thinking through narratives during their teaching practice experiences. Fernandez et al. (2020) also stated that examining prospective teachers' views on the feedback of their tutor and other prospective teachers in the discussion environment can also contribute to the field. In this context, this study aims to examine in detail the ability to notice student thinking in the process of narrative construction and discussion through the experience of a prospective teacher.

The primary objective of this study was to investigate the noticing skills of a prospective middle school mathematics teacher through the processes of narrative construction and discussion. To achieve this, the study aimed to reveal the level of change in noticing skills by sharing the narratives created by the prospective teacher as part of the Teaching Practice course and engaging in discussions and feedback within an online environment. In this context, the focus was on the prospective teacher's skills of attending to, interpreting, and responding to student thinking. The following research question was sought to be answered in the study: To what extent does the prospective mathematics teacher attend to, interpret, and respond to student thinking through narratives?

## 2 Method

### 2.1 Design

This research adopts a qualitative approach. Since the study aimed to explore a prospective teacher's ability to notice student thinking in depth, the qualitative research paradigm was chosen to examine the prospective teacher's experiences closely. Specifically, the case study design, a qualitative research method, was employed (Yıldırım & Şimşek, 2008). The focus of the case was the prospective teacher's noticing skills, which were analyzed through the narratives she had prepared and discussed.

### 2.2 Participants and procedure

The study was conducted in the spring semester of 2023-2024 within the scope of the Teaching Practice course. The prospective teachers who participated in the course were senior-year students in the Department of Elementary Mathematics Teaching. In this study, the prospective teacher whose noticing skills were examined was selected using critical case sampling, a type of purposive sampling method (Yıldırım & Şimşek, 2008). This approach allows the researcher to identify a critical case and conduct an in-depth examination of it. The prospective teacher was identified as a critical case due to her openness to professional development training and her



significant effort in creating narratives and contributing actively to discussions throughout the study. Her motivated participation in the process further reinforced her selection as a critical case. In this context, it can be inferred that the professional noticing skills of prospective teachers, as discussed in relation to this specific case, suggest that ‘if this prospective teacher encounters challenges with noticing skills, those who are less motivated and engaged in the process are likely to experience even greater difficulties.’ This makes the sampling method used critical case sampling (Yıldırım & Şimşek, 2008).

The prospective teacher was a female participant who voluntarily participated in the study. Within the scope of the Teaching Practice course, the prospective teachers spent at least 24 hours of teaching in internship schools during the spring semester, one day a week for six hours (Ministry of National Education [MoNE], 2018), and also attended the theoretical lectures of the supervisor lecturer within the scope of the Teaching Practice theoretical course for one semester.

The data of this study consisted of the written narratives prepared by the prospective teacher and her statements in the online discussions. Within the scope of the research, in order to analyze the prospective teacher’s ability to notice student thinking in the Teaching Practice course, the prospective teacher was asked to turn the points that she found important from the instructional situations she experienced during the internship practices into written narratives in the form of teacher-student dialogues, and feedback was given to her through online discussions in the light of these narratives. Accordingly, the prospective teacher shared the learning situations that she found significant in the lessons she taught in the internship schools during the semester with the researcher, who was the instructor of the course, in the form of narratives.

To further elaborate, in the initial phase of the study, narrative examples were provided to the prospective teachers by the researcher within the scope of the course. These examples were selected based on the work of Fernandez et al. (2020). During the narrative writing process, prospective teachers were instructed to: identify instances that reveal student thinking, interpret these instances by considering possible underlying reasons, and propose alternative ways of teaching.

In the online discussion environment, discussions were held through the narratives. In the online discussion environment, answers were sought, especially to the following questions: What do you notice about student thinking in the shared narrative? Why do you think the student might have thought this way? Did the prospective teacher notice the student's thinking in this narrative? If so, what kind of a path did he/she follow after he/she noticed it? What kind of a path would you follow? During the discussions, in addition to the prospective teachers giving feedback to each other, the researcher as the tutor also gave feedback to them in the light of guiding questions in the components of attending to, interpreting, and deciding on the next instructional action within the scope of the skill of noticing student thinking. Online discussions were held three times in total in the Zoom portal. The prospective teachers whose data were analyzed in the present study actively participated in all meetings. Within the scope of the study, a total of three narratives belonging to the participant's prospective teacher and discussed in online discussions were focused on. The first narrative of the prospective teacher (Melissa) is presented below as an example (Figure 1).

The dialogue related to the outcome “Draws a triangle given the dimensions of a sufficient number of elements”:

Melissa: Which elements do I need to know in order to draw a certain triangle?  
 Student 1: We need to know the side length.  
 Student 2: Angle measure.  
 Student 3: Edge-angle-edge, edge-edge-edge, angle-angle-edge.  
 (Students' answers are written on the board)  
 Melissa: Well, can I draw a triangle if I know only two angle measures?  
 Student 1: Yes, teacher.  
 Melissa: Now let everyone take their protractor and draw a triangle with one angle of 70 degrees and the other angle of 40 degrees.  
 Students: We drew it, teacher.  
 Melissa: How many different triangles can I draw according to these angle measurements, are all the triangles drawn the same?  
 Student 1: We can draw two.  
 Student 2: We can draw three. Because there are three angles, we can change their places.  
 Student 3: I think they will all be the same. Because the sum of the interior angles of all of them is 180, so the degrees are equal.  
 Melissa: Now, let everyone measure the perimeter of the triangle they drew and tell us what it is.  
 Students: 10, 8, 5, 4 ...  
 Melissa: So, how many triangles did we draw?  
 Students: More than one teacher.

(In the rest of the lesson, the students draw triangles whose length of three sides, length of one side and the measure of two angles, length of two sides and the measure of the angle between these sides are given respectively)

Figure 1 Melissa's first narrative

### 2.3 Data analysis

In this study, content analysis was employed to examine the prospective teacher's ability to notice student thinking through the components of attending to, interpreting, and responding. At this stage, the analytical frameworks developed by Jacobs et al. (2010) for noticing student thinking and by Fernandez et al. (2020) for prospective teachers' instructional decisions were utilized. The researcher began coding the data with these frameworks in mind, and the final framework was established after making necessary adjustments during the coding process (see Table 1).

Table 1 Themes and codes (Adapted from Jacobs et al., 2010 and Fernandez et al., 2020)

Themes	Codes		
Attend to	Proof of evidence	Partial NA*	Lack of evidence Not being able to identify important mathematical details, making only general statements, not understanding student thinking
	Robust Identifying important mathematical details		
Interpret	Making sense of and interpreting student thinking in different ways	Interpreting student thinking in less depth, commenting with general statements, making generalizations	Commenting with general statements that are not related to student thinking, giving positive evaluations, making instructional recommendations
	Taking student thinking into account, taking steps that can provide the benefits specified by the research, noticing		
Respond	Taking more general instructional steps rather than focusing on student thinking, asking the student		Taking steps that are not directed towards the student's thinking, continuing the lesson as

student thinking and going beyond the lesson plan, reminding prior learning, establishing connections, creating a basis by anticipating subsequent learning, making students discover, going to extensions	to explain his/her thinking more, explaining the mathematical meaning to the student	planned without considering student thinking, ignoring student thinking, proceeding by giving the direct answer
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\*Non-applicable

## 2.4 Validity, reliability, and ethical considerations

To ensure coding reliability, the researcher re-coded the data after finalizing the framework. Subsequently, an independent mathematics educator also coded the data individually. The researchers then compared their coding, achieving an inter-coder reliability of 88% (Miles & Huberman, 1994). According to Miles and Huberman (1994), inter-coder reliability above 70% is considered to indicate high reliability. The coders discussed the parts where there were code discrepancies, and as a result of these discussions, a complete consensus was reached. Additionally, to ensure the validity and reliability of the study, in the findings section, direct quotations were provided.

For the ethical considerations, the ethical permission was obtained by the university’s Social and Human Sciences Research Ethics Committee, and the prospective teacher voluntarily participated in the study. Within the scope of ethical rules, the identity of the prospective teacher is kept confidential, and she is represented under the pseudonym Melissa in this study.

## 3 Findings

The findings of the content analysis reveal that the prospective mathematics teacher (Melissa) predominantly demonstrated the ability to interpret and respond to student thinking and that she was able to do this to a partial extent.

**Table 2** The level of prospective teacher’s ability to notice student thinking in discussed narratives

Narrative# (N#)	Attend to		Interpret			Respond		
	Robust	Lack of evidence	Robust	Partial	Lack of evidence	Robust	Partial	Lack of evidence
N1	+		+				+	
N3	+			+			+	
N6	+		+				+	

When Table 2 is examined, it is seen that Melissa was able to identify student thinking in the attending component and that she did so at a robust level. Melissa was able to demonstrate a strong level of attending skill in three narratives that were also discussed in the discussions. In the interpretation component, while she was able to interpret at a robust level in two of her narratives, she remained at a partial level in one narrative. In the responding component, it was observed that Melissa could not reach the level of robust evidence. In all three narratives, Melissa remained at a partial level of evidence in the responding component. Examples of Melissa’s statements coded at different levels within the scope of each component are presented below.

Melissa’s third narrative, which was also discussed in Meeting 2, can be given as an example of the robust level of noticing skill in attending to the component. This narrative of Melissa is as follows (Figure 2):

The dialog that takes place while solving the question related to the outcome “Determines the similarity



ratio of similar polygons, forms polygons equal and similar to a polygon”:

Problem: If triangles ABC and DEF are similar, and the proportion of  $|AB|$  to  $|DE|$  is  $1/4$ , then what is the proportion of the perimeter of ABC to the perimeter of DEF?

The question above is given and everyone is asked to examine the question.

Melissa: What do you think the answer is?

Student 1: We are not given the side lengths; how can we find the perimeter?

Melissa: Draw the triangles ABC and DEF; maybe a solution will come to your mind.

Student 2: We can say 1 unit for side AB and 4 units for side DE.

Melissa: Very good, continue from here.

Students: How will we continue, can you give us a hint?

Melissa: We can say a unit for side AC and b unit for side BC. What are the side lengths of the triangle DEF?

Student 2:  $4a$  and  $4b$

Melissa: Now, proportion the perimeter lengths of the triangles as asked in the question.

(Students make solutions in their notebooks, and many students get help from their classmates)

Melissa: Did you find the answer?

Student 3: It was something very long. We must have made an operation error.

Melissa: Try putting the denominator in common parentheses, maybe there is a simplification.

Student 1: Yes, it became  $1/4$ .

Melissa: Yes, that's right. Now look at the similarity ratio given in the question.

Student 1: It is the same ratio.

Melissa: Then, do you think it would be right or wrong if I say that the proportion of the perimeter lengths of the polygons whose similarity ratio is given to us is equal to the similarity ratio?

Students: It would be right because we found it equal.

Melissa: Everyone should write this note in their notebooks.

(In the rest of the lesson, students solve a similar question and then try to find the ratio of the areas by using the similarity ratio given in the same way)

**Figure 1** Melissa's third narrative

When asked to define the situations that explain student thinking in relation to this narrative, Melissa shared the following statement:

Students first evaluated the given information to solve the problem. They tried to visualize the problem by drawing the triangles ABC and DEF, whose side lengths were not specified. Then, they calculated the perimeter lengths by symbolically determining the side lengths of the triangles. When they thought that there was an operation error in the solution of the problem, students realized the solution by simplifying it. Finally, they reached the correct result by comparing the perimeter lengths using the given similarity ratio.

Here, it was seen that Melissa was able to attend to student thinking at a robust level as she was able to identify important mathematical details.

Within the scope of the interpretation component, Melissa demonstrated robust interpretation skills in the first and sixth narratives, where she was able to make sense of and interpret student thinking in different ways. These narratives are among the narratives discussed in Meetings 1 and 3. For example, Melissa shared the following dialogue in her sixth narrative (Figure 3):

The dialog that takes place while solving the question related to the outcome “Forms the surface area relation of a right circular cylinder and solves related problems.”

Problem: One of the bases of a right circular cylinder with a radius of 5 cm is given. The height of this

cylinder is twice as long as its diameter. What is the lateral area of this cylinder in pi square centimeters?

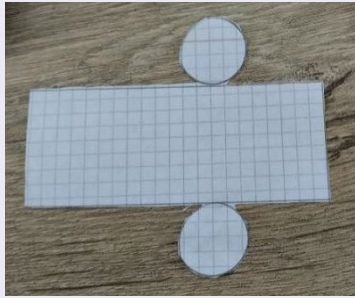
(The visual of the problem is opened on the smart board and everyone is asked to read the question)

Melissa: What is the answer?

Student1: We need to know the other length for the lateral area, but it is not given in this question.

Melissa: We can find the length with our previous knowledge. Now I will give you the open version of the right circular cylinder, and you think that the cylinder in the question is the cylinder in your hand. Write what is given in the question on the cylinder in your hand and fold it to form a cylinder.

(The material visualized below is distributed)



Melissa: Write what is given, and show where you want us to show by holding up the paper.

(Students show the lateral area)

Melissa: How else can I find the other length needed to find this area? Create the cylinder, and the side length is actually the perimeter length of another area.

Student2: Yes, the circumference of a circle.

Melissa: Yes, it can be found from the formula for the correct perimeter. What is the answer then?

Student1: 200

Melissa: Yes, that's right.

(In the rest of the lesson, the subject is reinforced by solving similar examples)

**Figure 3** Melissa's sixth narrative

In relation to this narrative, Melissa shared the following statement when she was asked to interpret the situations explaining the student thinking she had described by considering possible reasons:

The fact that students do not know the length required to find the lateral area at the beginning shows that they need better to understand the logic of surface area calculations and formulas. My - as the teacher- physical representation of the open cylinder helped students to understand these relationships visually and concretely. This approach enabled students to grasp geometry and area calculations in a more concrete way. The students' realization that the perimeter of the cylinder was a side of a rectangle showed that they understood the perimeter formulas and relationships. This process improved students' problem-solving skills and their ability to integrate mathematical concepts.

In her third narrative within the scope of the interpretation component, Melissa demonstrated partial interpretation skills. This narrative was discussed in Meeting 2. For this narrative, Melissa shared the following statement when she was asked to interpret the situations explaining the student thinking she defined by considering possible reasons:

There are several possible reasons behind students' successful problem-solving. First, students have the ability to visualize the problem and express it symbolically. Secondly, students collaborated in solving the problem, received help from each other and were able to find common solutions. Finally, students reached the correct conclusion by using the given

information and the similarity ratio effectively.

In this statement, it is understood that Melissa used general expressions and interpreted student thinking by generalizing rather than making detailed comments directly on student thinking.

It was observed that Melissa, who showed partial level of noticing skill in the responding component, remained at this level in her three narratives. For example, in the first narrative, Melissa asked her students the question, “Which elements would I need to know in order to draw a certain triangle?”. In line with the students’ answers, she asked the question, “Given two angle measures, how many different triangles can I draw? In other words, are all triangles drawn the same?”.

When asked what alternative ways she could offer to enhance learning in relation to this narrative, Melissa offered the following ideas:

**Visual and Auditory Processing:** Visual aids can be provided to students during triangle drawing. For example, by making an interactive triangle drawing on a projection or smart board (dynamic geometry software can be used. Geogebra, Google Classroom, etc.), the steps can be explained visually. In addition, videos on triangle drawing can be shown, or digital materials can be presented to help students understand the topic better.

**Student-Centered Activities:** The learning process can be enriched by providing students with interactive activities related to triangle drawing. For example, students can be asked to draw different triangles in groups and discuss the properties of these triangles. In addition, a triangle drawing contest can be organized with students according to certain criteria.

**Math Games:** Students can be motivated by playing math games related to triangle drawing. For example, students can practice drawing triangles using puzzles or math card games.

**Visual Math Tools:** Geometry sets can be used to help students draw geometric shapes and triangles. Students can use tools such as rulers, compasses, and protractors to draw triangles for a more effective learning experience.

**Project-Based Learning:** The topic can be made more meaningful by having students work with real-life examples of triangle drawing. For example, students can be asked to research how triangles are used in architecture, engineering, or art and prepare presentations. In this way, we can ensure that students have the opportunity to apply their math skills in a real-world context.

In Meeting 1, Melissa again argued that in terms of responding to student thinking, it would be useful for students to experiment by drawing triangles on applications such as Geogebra or Google Classroom that can be opened from the smart board and to make observations by lengthening and shortening the side lengths and/or changing the angles of the triangle. On the other hand, she added that she could not experience these alternative ways of responding in her internship due to reasons such as internet problems or the necessity to move on to the next learning outcome.

In Meeting 2, where a discussion was held on Melissa’s third narrative, it was observed that Melissa offered various alternatives for the responding component (while it was at partial level in her narrative) during the discussions. Melissa stated that making sense of the rule through visualization by using applications such as demos and Geogebra, asking students to create triangles and reach their own conclusions by making measurements, and/or creating environments on the geometry board where students can reach generalizations from similarity ratios themselves would increase student understanding. Similarly, she stated that these steps could also be carried out for the areas of triangles. Thus, Melissa again displayed partial responding skills in the discussions.

In Meeting 3, it was observed that Melissa was able to respond strongly to the narrative of another

prospective teacher who exhibited partial responding skills. During the discussions, Melissa argued that it would be beneficial for the student to visualize the expansion of the cylinder through simulations that can be made available to the student (by using GeoGebra or ready-made videos on YouTube) and that opening and closing the object and seeing that the circumference of the circle actually corresponds to the length of a side of a rectangle would help the student grasp the logic of the problem.

In addition to these findings, different points emerged during the discussions regarding Melissa's level of responding to student thinking. For example, during Meeting 1, in which another prospective teacher's narrative was discussed, Melissa responded to the question "What would you have them do in the next stage, how would you proceed?" by saying:

The student thought directly based on the formula without associating the old knowledge with the new ones. I would create the diagonals of a rhombus and have them discover that the intersection point centers the diagonals by making them measure. They need to see and discover for themselves that the diagonals center each other and divide the rhombus into congruent triangles. Here, she thinks that she can solve the problem only by using the area formula. She cannot make associations.

Here, it was observed that Melissa exhibited a robust level of responding skills by recalling prior learning, establishing associations, and building a foundation for student understanding.

Regarding the responding component, in Meeting 1, it was also observed that during the discussion about a narrative, other prospective teachers defended the idea of having the students do the multiplication of ins and outs. On the other hand, only Melissa objected to this. Melissa stated that in such cases where students have difficulty solving equations, they can apply the same operation to both sides of the equation. Melissa argued that the student could succeed in leaving  $x$  alone by reminding the inverses of the operations. She stated that the only way to solve such equations is not the product of ins and outs and that the logic of the actual operation can be handled in this way. After that, it was observed that other prospective teachers stated that they would only prefer to use this method if the student knew the inner-outer product and wanted to use it but could not remember it.

Furthermore, regarding the responding skill, it was observed that a prospective teacher who was at the level of lack of evidence in her narrative was inspired by Melissa's responding steps and stated that she thought it would be effective to concretize in situations where students need to make an inference about the ratio of perimeter lengths based on the ratio of side lengths in similar triangles. Similarly, another prospective teacher (at a partial level in her narrative) stated that she could concretize the subject by teaching the lesson through the use of GeoGebra in terms of responding skills.

Another point is that in Meeting 3 when Melissa took the floor on her sixth narrative, she stated that she realized that her prompt, "Form the cylinder, the length of the side is actually the perimeter length of another region." was overly directive, that she probably did this due to time constraints, and that she thought it would be better to give students more time and let them realize/discover this themselves.

In addition, there were cases where Melissa took her noticing skills one step further with the guidance of the tutor during the discussions. For example, in a narrative containing a problem situation requiring the calculation of the area of a right trapezoid, when steps such as reminding the area formula of the trapezoid or calculating the area of the trapezoid by dividing the trapezoid

into a rectangle and a triangle were suggested in response to the student who chose to add the base lengths and multiply them by the height, the tutor asked, “Well, can the student be guided by asking him to establish an association with the area formula of the rectangle?”, Melissa realized that this would strengthen the student’s conceptual understanding and that she could easily proceed by using this reasoning in similar situations she would encounter in the future. At this stage, Melissa was able to exhibit higher-level responding skills by explaining how she would continue the lesson in this context.

## 4 Discussion

Teachers must be attentive to students’ thought processes, design opportunities for engaging with these thoughts, listen carefully to students’ ideas, and adapt their instruction accordingly (Ivars et al., 2020; Monson et al., 2020). However, as previously noted, noticing is a skill that is not inherently acquired (Jacobs et al., 2010; van Es & Sherin, 2008), and thus professional development contexts are essential for cultivating this ability. It is also important to recognize, as El Mouhayar (2024) emphasizes, that the development of noticing skills, particularly in prospective teachers, may present challenges and can require a significant amount of time.

In this study, which aimed to examine a prospective teacher’s noticing skills toward student thinking through narratives, the findings indicated that the prospective teacher was able to exhibit a robust level of attending skill; she exhibited both a robust and partial level of interpreting skill but mainly exhibited partial responding skill. These findings are in line with some studies in the literature. For example, the findings of Toluk Uçar and Bozkuş’s (2023) study indicated that prospective teachers’ ability to notice student thinking was higher in attending to the component but insufficient in the interpreting and responding components.

Regarding the skill of responding to student thinking, the findings of this study revealed that the prospective teacher was unable to demonstrate this skill at a robust level in the narratives, remaining at a partial level of evidence. This aligns with previous research suggesting that teachers often struggle with responding to student thinking (Franke et al., 2009). Specifically, teachers may face challenges in asking effective questions, fostering student reasoning without being overly directive, and emphasizing conceptual understanding over procedural methods (Son, 2013). Several studies have highlighted the difficulty of responding effectively to student thinking. For instance, a recent study by Çopur and Tekin Sitrava (2024) concluded that prospective teachers exhibited low noticing skills in the responding component, underscoring the need for targeted professional development in this area of noticing.

On the other hand, in the present study, during the online discussions, it was observed that the prospective teacher showed some improvement, especially in the responding component of noticing skills. In an environment where ideas were discussed collectively, there were situations that showed that the prospective teacher was able to improve her responding skills to a higher level. It is understood that the prospective teacher was able to benefit from the ideas of other prospective teachers in the group and also from the guidance of the tutor. In this context, it can be thought that discussion opportunities through narratives can contribute to the prospective teachers’ ability to notice student thinking by benefiting from the ideas of both each other and their tutors. As Schack et al. (2013) state, providing prospective teachers with environments where they can use their skills of attending to, interpreting, and responding to student thinking contributes to the development of their professional noticing skills. The findings of Ivars and Fernandez (2018) are consistent with this, as they emphasized that prospective teachers require



guidance from their tutors to enhance their noticing skills, which, in turn, can improve their instructional decision-making. In the current study, the fact that the prospective teacher showed improved response skills by drawing on ideas shared during discussions highlights the potential value of offering such professional development opportunities for prospective teachers.

#### 4.1 Limitations and future directions

The present study has some limitations. In this study, a total of three narratives of one prospective teacher in three online discussion environments were included. In future studies, more narratives can be analyzed in discussions with a larger number of prospective teachers to examine how prospective teachers show skills at different times and in different narrative contents. By conducting more discussions, it can be analyzed whether there is an improvement in the prospective teachers' noticing skills from the beginning to the end of the process. At this point, as El Mouhayar (2024) points out, the development of noticing skills may depend on different factors beyond time, such as individual characteristics or classroom context. In this context, it may be suggested that other possible factors that may have an impact on the process should be taken into consideration when examining the development of noticing skills through narratives in future studies. In addition, as in Fernandez et al.'s (2020) study, the changes and developments in prospective teachers' narratives can be examined, and the relationship between the feedback provided in the online environment and these changes can be revealed. It may be possible to obtain richer content by recording the discussions face-to-face rather than online.

#### 4.2 Conclusion

In summary, the literature suggests that various learning environments designed for the professional development of both in-service and prospective teachers can enhance their noticing skills. Methods such as analyzing student thinking through examples (Walkoe, 2015) and engaging in group discussions (Sherin & Han, 2004) are among these approaches. Additionally, analyzing student thinking through narratives (Fernandez et al., 2020) is another practice that can support professional growth. As El Mouhayar (2024) highlights, there is a need for a deeper understanding of the role narratives play, given their pedagogical importance, in fostering the development of noticing skills. Therefore, further research is required to explore prospective teachers' noticing skills through the use of narratives. As demonstrated in this study, narratives might reveal the noticing abilities of teachers regarding student thinking, and these skills can be further developed through group discussions and shared insights.

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