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Reflections from learning activities designed by prospective teachers and supported by mathematical process skills

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Abstract

This study aims to examine the processes by which prospective teachers design mathematical activities based on mathematical process skills (connections, communication, and reasoning) and in line with the constructivist learning approach. The study was conducted within the scope of the course titled "Connections in Mathematics Teaching," offered in the primary mathematics teacher education program at a public university. A case study design, a qualitative research method, was adopted. The study group consisted of 55 prospective mathematics teachers enrolled in the aforementioned course. Data were collected through activity reports prepared by the participants, observation notes recorded by the researcher during the activity presentations, and group interviews conducted at the end of the process. The collected data were analysed through content analysis within the framework of pedagogical and skill-based dimensions. The findings indicate that the prospective teachers were mainly able to integrate mathematical process skills into their activities and had reached a satisfactory level, particularly in the areas of making connections and mathematical communication. However, significant deficiencies were observed in their ability to identify and utilize indicators related to 'estimation-based reasoning.' From a pedagogical perspective, it was found that the participants demonstrated limited performance in the exploration and explanation phases, which constitute key components of constructivist teaching. In light of these results, it is recommended that teacher education programs incorporate a greater number of constructivist, practice-oriented, and reflective activities to support the pedagogical development of prospective teachers.

Keywords: Constructivist learning, pedagogical analysis, connections, communication, reasoning.

Introduction

Although mathematics is often perceived by many students as a field detached from real life due to its abstract nature, in reality, it is a fundamental discipline utilised across a broad spectrum, from everyday experiences to complex scientific endeavours. While it is expressed through numbers and symbols, mathematics is far more than mere calculations; it is a powerful tool that systematises thinking, reasoning, and problem-solving. In this regard, mathematics not only fosters individuals' logical reasoning abilities but is also effectively applied in numerous disciplines, ranging from the natural sciences to economics, technology, and the social sciences. What renders mathematics indispensable is its functionality as a tool for solving problems encountered in various aspects of life.

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Across the world, mathematics education aims to convey the nature and utility of this discipline to students in an effective manner. Accordingly, a range of approaches, standards, and instructional principles have been developed to enhance the quality of mathematics education. One of the leading institutions guiding this process is the National Council of Teachers of Mathematics (NCTM), based in the United States. Since the 1980s, the NCTM has played a significant role in shaping mathematics curricula in developed countries through its reports and teaching standards, offering a framework for structuring teachers' instructional practices. In order to ensure that mathematics teaching becomes more effective and student-centred, the NCTM has proposed standards in two key domains, tailored to different grade levels, content standards and process standards. These standards aim not merely for the rote memorisation of mathematical facts but rather for students to comprehend and apply mathematical knowledge meaningfully. The content standards encompass the main domains of mathematics, *(i) number and operations, (ii) algebra, (iii) geometry, (iv) measurement, and (v) data analysis* and are designed to support balanced development in these core areas. The process standards, on the other hand, focus on how students learn mathematics and include essential mathematical competencies that students are expected to develop. These skills are expressed as *(i) problem solving, (ii) reasoning and proof, (iii) communication, (iv) connections, and (v) representation* (NCTM, 2000). These competencies are not limited to classroom achievement; rather, they represent essential skills that enable students to integrate mathematics with real-life contexts. Abilities such as solving problems, reasoning, expressing mathematical ideas clearly, using multiple representations, and establishing interdisciplinary connections help students apply mathematics effectively in authentic situations.

The mathematical process skills proposed by NCTM have had an impact not only at the international level but also on mathematics curricula in Turkey. In particular, with the renewal of primary education programmes in 2005, Turkey adopted the constructivist learning approach, placing emphasis on a student centred and meaning-oriented instructional model (Ocak, 2020; MoNE, 2005). This transformation marked a shift from a knowledge transmission model to one that aims for students to learn through active participation, exploration, and inquiry. For the first time, the revised curriculum explicitly included four mathematical domain skills, *i) problem solving, ii) making connections, iii) communication, and iv) reasoning* (Karabey & Erdoğan, 2023; MoNE, 2005). These skills have enabled students to construct mathematical knowledge, relate it to daily life, and retain what they have learned more permanently. Thus, mathematics instruction has evolved into a more functional structure, aiming not only at conceptual understanding but also at applying knowledge to real-life situations. Among these, reasoning, problem solving, communication, and making connections are regarded as essential foundational skills in mathematics education (Baykul, 2009). In this context, the current Turkish mathematics curriculum highlights *i) communication, ii) making connections, and iii) reasoning* as key process skills, emphasising their acquisition as fundamental competencies within school mathematics (MoNE, 2013). These process skills are also widely recognised in the literature (Kaur & Lam, 2012; NCES, 1999; NCTM, 2000; Van de Walle et al., 2015) as integral components of mathematical working processes.

Communication

One of the mathematical process skills, mathematical communication, is defined as "the ability to express, understand, interpret, and evaluate mathematical ideas both in written and verbal form; to represent ideas using different models; and to use terms, notation, and mathematical structures to explain the relationships between these mathematical models" (Rajagukguk, 2016, as cited in Kıymaz et al., 2020, p. 206). Kaya and Aydın (2016) describe mathematical communication as structured interactional activities conducted in the classroom, including strategies such as questioning, discussion, and group work. In this context, the aim of mathematical communication is to encourage students to express, share, and reflect on their ideas (Aydın & Author, 2025). In the NCTM (2000) document, mathematical communication is described as a fundamental skill used in classroom environments where learning is actively constructed by the individual. In order for students to acquire this skill, they need to develop the skills of "(i) organize and consolidate their mathematical thinking through communication, (ii) communicate their mathematical thinking coherently and clearly to peers, teachers, and others, (iii) analyze and evaluate the mathematical thinking and strategies of others, and (iv) use the language of mathematics to express mathematical ideas precisely" (NCTM, 2000, p.268). In the 2013 Turkish mathematics curriculum, the indicators of communication skills are outlined as; i) recognising that mathematics is a language with its own unique symbols and terminology; ii) using mathematical symbols and terms effectively and accurately; iii) applying mathematical language appropriately and effectively within mathematics itself, across different disciplines, and in daily life; iv) expressing mathematical ideas through various forms of representation such as concrete models, shapes, images, graphs, tables, and symbols; v) expressing mathematical thinking both orally and in written form; vi) relating everyday language to mathematical language and symbols, and vice versa; and vii) interpreting the accuracy and meaning of mathematical thinking" (MoNE, 2013, p. v). Finally, in the 2024 Education Model curriculum, mathematical communication is included both as a process component of mathematical representation skills and as one of the social-emotional learning competencies (MoNE, 2024, as cited in Aydın & Author, 2025).

Connections

The skill of making connections, recognised both in national and international curricula as an integral part of mathematics learning processes, is identified by NCTM as one of the essential mathematical skills that students should acquire. According to NCTM, the indicators of this skill, applicable from early childhood through to the end of Grade 12, include "(i) Recognize and use connections among mathematical ideas, (ii) Understand how mathematical ideas interconnect and build on one another to produce a coherent whole, and (iii) recognize and apply mathematics in contexts outside of mathematics" (p. 274). In the Turkish mathematics curriculum (MoNE, 2013), the development of students' connection-making skills is aimed through the following behaviours: "(i) establishing relationships between concepts and operations, (ii) representing mathematical concepts and rules using different forms of representation, (iii) connecting and transforming different representations of mathematical concepts and rules, (iv) connecting various mathematical concepts to one another, and (v) connecting mathematics with topics and situations encountered in other subjects and daily life" (p. vi). Finally, the Türkiye Century Education Model, published and implemented in 2024, introduces a new skill referred to as "building bridges" instead of the traditional notion of making connections. In this framework, building bridges is defined as the process of forming links between students' existing knowledge and skills and the new knowledge and skills they are expected to acquire. It is further described as the connection of classroom learning with real-life contexts (MoNE, 2024, as cited in Aydın & Author, 2025).

Reasoning

According to NCTM (2000), reasoning is an integral part of doing mathematics and should be embedded within the curriculum at every educational level, from early childhood through the end of secondary education. In the relevant document, the indicators of reasoning are outlined as: "(i) recognize reasoning and proof as fundamental aspects of mathematics, (ii) make and investigate mathematical conjectures, (iii) develop and evaluate mathematical arguments and proofs, and (iv) select and use various types of reasoning and methods of proof (NCTM, 2000, p. 262). In the Turkish mathematics curriculum (MoNE, 2013), the indicators of reasoning skills are listed as follows: "(i) defending the validity and accuracy of inferences, ii) making logical generalisations and inferences, iii) explaining and using mathematical patterns and relationships when analysing a mathematical situation, iv) making estimations about the results of operations and measurements using strategies such as rounding, grouping appropriate numbers, or focusing on the leading or trailing digits, as well as self-developed strategies, v) making measurement estimations based on a specific reference point" (p. v). Lastly, in the Türkiye Century Education Model implemented in 2024, mathematical reasoning is identified as one of the five core domain skills and is composed of four main sub-skills: (i) analysing, (ii) interpreting, (iii) making inferences, and (iv) conducting mathematical validation and/or proof (MoNE, 2024, as cited in Aydın & Author, 2025).

Mathematical activity in terms of constructivist approach

The constructivist teaching approach is an instructional model in which students actively construct knowledge, relate new learning to their prior experiences, and engage in meaningful learning processes. This approach encourages students to take an active role in their own learning and to acquire knowledge through exploration and discovery. Consequently, the role of the teacher also shifts: rather than being a transmitter of information, the teacher becomes someone who facilitates students' access to new knowledge by building upon their prior knowledge and lived experiences. In this context, the teacher acts as a guide, facilitator, or mentor who supports and scaffolds students' learning (Hoagland, 2000; Rita, 2002). In order to implement the constructivist teaching approach effectively, it is recommended that lessons follow a structured sequence of phases. Among the various learning activity models, the 5E Learning Cycle Model developed by Rodger Bybee is one of the most widely used. This model consists of the following stages: "Engage", "Explore", "Explain", "Elaborate", and "Evaluate" (Bybee et al., 2006). The Engage phase aims to capture students' attention and activate their prior knowledge through the use of questions, short videos, or real-life examples. This stage stimulates students' curiosity and motivates them to learn. In the Explore phase, students investigate new concepts through group work, experiments, or problem-solving activities. During this process, the teacher facilitates learning by guiding students and supporting them as they construct knowledge through their own experiences. In the Explain phase, students share and make sense of the experiences they gained during the exploration stage. The teacher listens to students' explanations, clarifies concepts when necessary, and reinforces learning by introducing appropriate scientific terminology. During the Elaborate phase, students apply the concepts they have learned to different contexts. By connecting their knowledge to real-life problems, they deepen their understanding. In the Evaluate phase, students' learning processes and outcomes are assessed. The teacher employs various assessment tools to measure students' levels of understanding and provides constructive feedback.

In the curriculum revised in line with the constructivist learning approach, emphasis is placed on instruction through activities, and it is stated that activities play a significant role in the success of the programme. Uğurel and Bukova Güzel (2010) define the concept of 'activity' as a learning or working action initiated (willingly) through the interaction between the individual and their environment. Özmantar et al. (2010) summarise the key aspects of the activity concept as educational tasks that involve active student participation through the use of tools and resources, require students to take responsibility, aim to produce a particular learning outcome, and are designed to be engaging and curiosity-inducing. In some studies within the field, a discipline-specific interpretation of the activity concept is emphasised, such as the notion of 'mathematics learning activity' in mathematics education. Synthesising all studies based on mathematics learning activities (MLAs), Toprak et al. (2017) define a mathematics learning activity as follows:

> A Mathematics Learning Activity (MLA) is defined as a student-centred structure that is designed in accordance with the constructivist learning approach, allowing individuals to construct mathematical knowledge by establishing connections with real-life contexts, other disciplines, and topics within mathematics itself. It facilitates learning and teaching, can be conducted through group work, requires diverse thinking and creativity, and incorporates mathematical process skills such as abstraction, inference, mathematical thinking, problem-solving, reasoning, and modelling. Moreover, it necessitates the use of mathematical symbols, demonstrates the continuity of mathematics, and is characterised by being engaging, systematic, and well-planned, with applications carried out within the classroom setting (p. 10–11).

An examination of the relevant literature reveals the existence of various frameworks concerning the essential characteristics that mathematical activities should possess when developed through a constructivist approach. Brooks and Brooks (1993) asserted that the problems to be solved and the activities to be discussed in the learning environment should be selected in a way that captures students' interest. Suzuki and Harnisch (1995) outlined several features that mathematical activities should include, such as incorporating real-life situations, offering multiple solution paths, demonstrating the continuity of mathematics rather than isolated structures, and enabling students to develop conceptual understanding through communication. Elçi et al. (2006) emphasised the necessity of establishing connections with daily life, other scientific disciplines, and prior knowledge when designing activities. In addition to the structural characteristics of activities, some researchers (e.g. Coşkun, 2005) highlighted the importance of organising activities in a student-centred manner that ensures active learner engagement during the learning process. Furthermore, Baki (2008) underlined that activities should arouse curiosity, and the desired concepts, relationships, and properties should be embedded systematically and appealingly within the activity. He also emphasised that activities should engage students in cognitive processes such as the use of mathematical expressions and modelling, logical reasoning, the application of mathematical symbols, and abstraction. In the Ministry of National Education's (MoNE, 2011) mathematics curriculum, it is stated that the activity-based learning approach aims to equip students with a range of mathematical competencies and skills. Furthermore, it is emphasised that, in addition to active student participation in the implementation process of these activities, students are also expected to develop competencies such as mathematical thinking, problem solving, making connections, using mathematics as a language of communication, and modelling.

Olkun and Toluk (2005) present a more detailed framework by outlining the key components of a mathematics activity aligned with the constructivist learning approach: (i) intuitive phase, (ii) structured activity, (iii) discussion–explanation, (iv) concept/rule formation, (v) application and evaluation. In the intuitive phase, students' attention is drawn to the concept through a question or problem, encouraging them to reflect on it. This is followed by a structured activity related to the concept, during which students are expected to work in groups, engage in discussion, and generate questions. The activity may involve experiments with concrete materials, conducting measurements, or problem-solving using visual representations. In the discussion–explanation phase, students are encouraged to reflect on their previous actions, discuss them with peers, and share their findings. Subsequently, students are guided to derive generalisations from their experiences and collectively evaluate their validity, discussing any misconceptions. In the application phase, students adapt their learning to new situations or problems. Although the evaluation phase is considered the final step, formative assessment and process evaluation throughout the activity implementation are emphasised. These phases, when aligned with the components of the 5E instructional model, can be said to correspond as follows: the intuitive phase aligns with the 'engage' stage; the structured activity phase with the 'explore' stage; the discussion–explanation phase with the 'explain' stage; and the application and evaluation phase with the 'evaluate' stage. In line with this theoretical alignment, the processes to be used as the basis for the research design were systematically identified.

The significant role of mathematical learning activities in the teaching and learning of mathematics has been highlighted in numerous studies (Chapman, 2013; Simon & Tzur, 2004; Stylianides & Stylianides, 2008). The NCTM (1991) document states that activities "provide the stimulus for students to think about particular concepts and procedures, their connections with other mathematical ideas, and their applications to real-world contexts" (p. 24). Furthermore, it can be argued that the effectiveness of the implementation process of these activities largely depends on the knowledge and skills of teachers (Chapman, 2013; Karakuş & Yeşilpınar, 2013). Sullivan et al. (2009) emphasise that in order to effectively design activities that require high-level mathematical process skills such as reasoning and making connections, teachers must possess the necessary knowledge and competence. Therefore, it is essential for prospective mathematics teachers to attain a certain level of knowledge, skills and experience related to the design and implementation of such activities (Özgen & Alkan, 2014). Based on this premise, the present study investigates the process by which prospective teachers design mathematical activities within the framework of mathematical process skills and the constructivist learning approach.

Method

This study was conducted using a qualitative research design, adopting the case study method. A case study is a qualitative research method that aims to examine a specific situation or phenomenon in depth and from multiple perspectives (Merriam, 2013; Yin, 2018). In this method, the researcher defines the boundaries of the case and tries to understand it in detail within its own context. Given that this study explores prospective teachers' processes of designing learning activities based on the constructivist approach, the case study method was deemed appropriate for gaining a deep understanding of this complex and context-dependent phenomenon. The case study approach is frequently preferred for analysing such design-based instructional processes, as it allows for a detailed exploration of participants' experiences, thoughts, and decision-making processes (Creswell, 2013).

Participants and procedure

The data for this study were collected during the 2024–2025 academic year as part of the course 'Connections in Mathematics Education' offered in the Primary Mathematics Teacher Education undergraduate programme at a state university. In the first seven weeks of the 14-week course, prospective teachers were introduced to key competencies highlighted in mathematics education, which were examined in detail in line with the standards of the NCTM and the indicators included in the Turkish mathematics curriculums. In the following two weeks, theoretical and practical guidance was provided on how to integrate these competencies into mathematics lessons, and various sample activities were used to support participants' design skills. In the final four weeks of the course, prospective teachers were asked to design mathematical learning activities that incorporated indicators of reasoning, connections, and communication skills. During this stage, the class was divided into 11 groups of five students, and each group was tasked with enacting their designed activity in a classroom setting. As a result, each group had the opportunity to present and evaluate their activity during the final four weeks of the course. A total of 55 prospective teachers participated in the study. For participant selection, convenience sampling was used, a method preferred for practical reasons such as time, cost, and accessibility, involving individuals or situations that the researcher can easily reach (Yıldırım & Şimşek, 2011). Participants were informed that the data obtained during the course would be used for scientific purposes, and voluntary consent was obtained for their participation in the research.

Data collection tools

The data collection tools of the study consisted of (i) activity reports prepared by the prospective teachers, which included detailed information on the learning outcomes, objectives, duration, and implementation steps of the designed activity, (ii) observation notes recorded by the researcher during the in-class presentations of the activities, and (iii) group interviews conducted with the prospective teachers. In the activity reports, the design process was described in detail, and all interactions between the teacher and students within the activity scenario, including dialogue, were explicitly articulated. In addition, the group interviews focused on clarifying issues that emerged during the data analysis process concerning the designed activities. Each interview was conducted with the respective group of prospective teachers, and the information shared by participants during these interviews was noted by the researcher and incorporated into the data analysis.

Data analysis

The data analysis process of this study was structured in line with the key processes identified through the theoretical alignments established within the research framework and was carried out in two stages, the 'skills dimension' and the 'pedagogical dimension'. Accordingly, the lesson processes designed by the prospective teachers were examined under three components, i) introduction, ii) structured activity (exploration), and iii) explanation (class discussion). In both dimensions, content analysis was employed, and the collected data were categorised under specific themes based on similarities and differences (Yıldırım & Şimşek, 2011).

Within the skills dimension, the focus was placed on the extent to which the prospective teachers appropriately utilised the indicators of mathematical process skills (MoNE, 2013) in the learning activities they developed. If the described instructional process fully addressed a given indicator, it was coded as 'appropriate'; if it partially reflected the indicator, it was coded as 'partially appropriate'; and if it did not correspond to the indicator at all, it was coded as 'inappropriate'. Based on these criteria, the performance of the prospective teachers was evaluated under three distinct categories.

The data analysis conducted within the pedagogical dimension was carried out in alignment with the core instructional phases. In the introduction phase of the activity, prospective teachers were expected to plan and implement preparatory learning experiences that would enhance students' interest in the lesson, direct their attention to the topic, and activate their prior knowledge. In the structured activity (exploration) phase, they were expected to design learning environments that would promote students' active engagement in the learning process and enable them to construct and discover knowledge through experience-based activities, while also developing pedagogically sound strategies to guide this process. Finally, in the explanation (class discussion) phase, prospective teachers were expected to facilitate the sharing of experiences gained during the exploration phase through class discussion and to help students consolidate their learning by making sense of these experiences. In the analysis of all these phases, the performance of each prospective teacher was evaluated by categorising it as appropriate, partially appropriate, or inappropriate, based on the extent to which their performance met the expected objectives of the activity. In the presentation of the findings, the coding processes and results pertaining to the skill dimension have been specifically emphasised through the use of italicised text. Student quotations were coded using the abbreviation 'S', while teacher quotations were coded using 'T', each accompanied by a number for identification purposes. In addition, in the presentation of the findings related to the use of process skills by different groups included in the study, the abbreviations 'Gi(j)' were used. Here, 'I' represents the group number, while 'j' indicates the frequency with which the corresponding indicator was used.

Validity, reliability, and ethical considerations

In order to ensure the validity and reliability of the data analysis process conducted within the scope of the research, expert opinions and inter-coder reliability were utilised. Accordingly, all coding procedures were re-coded by the researcher at the end of the study. In addition, all data analysis procedures were independently conducted by another expert in mathematics education. At the end of the process, the consistency coefficient between the two coders was calculated as 0.94 for the skill dimension and 0.90 for the pedagogical dimension (Miles & Huberman, 1994). According to Miles and Huberman (1994), inter-coder reliability above 70% is considered to indicate high reliability. Based on this, it was accepted that the obtained values demonstrated the reliability of the coding process. During the finalisation of the data analysis, any disagreements between the coders were discussed and consensus was reached. Moreover, in support of the validity and reliability of the study, detailed descriptions of the activity design processes created by the prospective teachers were presented in the findings section.

Findings

In the findings section of the study, the coding processes related to the skill-based and pedagogical dimensions of the activities designed by different groups were presented in detail, supported by direct quotations. In the subsequent sections, the overall findings obtained from the study were conveyed in a descriptive manner.

Findings derived from Group 1

In the study, the first group of prospective teachers designed an activity addressing the learning outcome “MAT.5.3.7. To be able to reason about triangles constructed using the centres and one of the intersection points of a pair of intersecting circles on a plane, with the aid of mathematical tools and technology” (MoNE, 2024, p.39). The group members stated the objective of the activity as follows: ‘We aim for students to construct triangles using a pair of circles and to learn to classify these triangles according to their sides.’ In the introduction phase of the activity, the teacher draws the students’ attention to the concept of triangles and asks them to provide examples of triangles from everyday life.

Engagement.phase

> T: Children, when you look around, what kinds of triangular shapes do you see or come across?
>
> S1: Teacher, this morning on my way to school, I saw some triangular traffic signs from the minibus.
>
> S2: Teacher, I went to Egypt with my family last summer, and I noticed that the pyramids looked like triangles.
>
> S3: I also went camping with my dad last weekend, and I realized that the tent we set up looked like a triangle too.
>
> T: Those are all great examples, children. Let me give you a few examples as well.

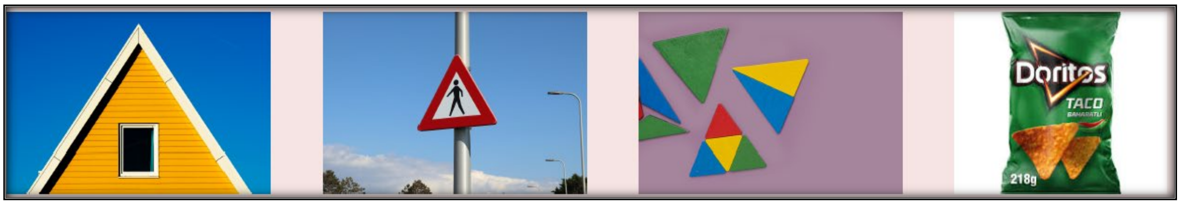

Figure 1 The visuals used in the introduction stage of Group 1’s activity

In the introduction stage of the activity, it was observed that the prospective teachers began the lesson in a way that would capture the students’ interest. Therefore, the performances of the prospective teachers at this stage were considered appropriate.

In this part of the activity, the prospective teachers stated that they employed the indicator of the connecting skill, namely ‘connecting mathematics with topics and situations encountered in other subjects and daily life’. They also reported using the indicators of the communication skill defined as ‘expressing mathematical ideas through various forms of representation such as concrete models, shapes, images, graphs, tables, and symbols’ and ‘relating everyday language to mathematical language and symbols, and vice versa’. The first indicator of the connecting skill, as expressed by the prospective teachers, was coded as ‘appropriate’, since it involves relating a mathematical concept to situations encountered in daily life. Secondly, the indicator related to the communication skill was also coded as ‘appropriate’, as it entails representing a mathematical concept through concrete models. However, it can be stated that the visuals from everyday life used to illustrate the concept of triangles do not fully reflect the intended indicator. This is because the indicator refers to students being able to explain real-life situations using mathematical concepts, terms, or symbols, or to make sense of mathematical expressions by connecting them to everyday contexts. The act of merely showing triangle-related visuals from daily life was therefore not considered sufficient to meet the criteria of this indicator and was coded as ‘partially appropriate’.

Structured.activity.(exploration).phase

In this phase, the initial dialogue between the teacher and the students proceeds as follows:

> T: Children, in our previous lessons we learned how to construct triangles. Do you remember how we did it?
>
> S1: I remember, teacher. We used to draw intersecting lines.
>
> T: Please come to the board and draw it so your classmates can see. (S1 comes to the board and draws the figure.) Yes, children, your friend has drawn it correctly. Now, do you think we can construct a triangle in a different way?
>
> S2: But teacher, how else can we draw a triangle?
>
> T: For example, can we construct a triangle using two circles? Give it a try. Would you like to come to the board and try? (The teacher turns to S2).

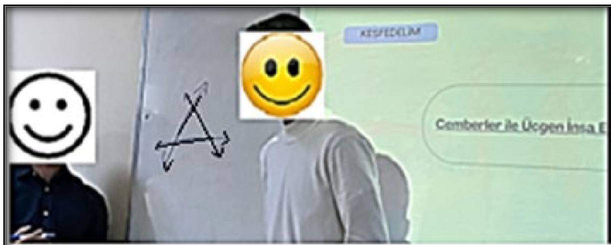

Figure 2

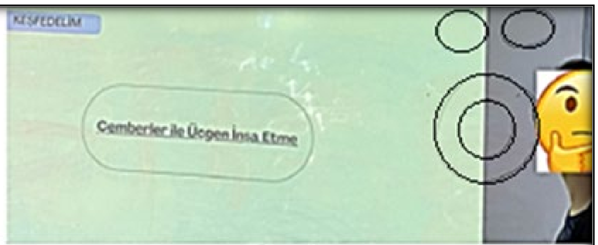

Figure 3

Here, the students successfully constructed a triangle using intersecting lines on the board (Figure 2); however, they were unable to construct a triangle using circles (Figure 3).

In this section, the prospective teachers stated that they employed the reasoning indicator ‘making estimations about the results of operations and measurements using strategies such as rounding, grouping appropriate numbers, or focusing on the leading or trailing digits, as well as self-developed strategies’. However, since there was no explicit mathematical operation or measurement involved and the students were more engaged in a trial process rather than actual estimation, this indicator was coded as ‘partially appropriate’.

At this stage, it is noteworthy that the students did not initially consider the case involving the intersection of two circles. As the process continued, the teacher stated, ‘Let’s do it together then, children. I’m drawing a circle on the board and marking its centre. Then, I draw another circle that intersects with the first one and mark its centre as well. Finally, I connect the centres of the two circles with one of their points of intersection,’ and demonstrated the construction on the board. When the students responded with expressions of surprise such as ‘Wow, teacher, how did you do that?', the teacher encouraged them to try it themselves by saying, 'Come on then, come to the board and give it a try.'

In this part of the activity, it is noticeable that the teacher did not fully allow the students to explore independently and were unable to provide adequate guidance. Within the context of constructing triangles using intersecting circles, the curriculum states that "students are expected to make conjectures about the side properties of triangles constructed using the centres and one of the intersection points of two intersecting circles" (MoNE, 2024, p. 45). Therefore, it could be argued that a more appropriate approach would have been for the teacher to prompt students to consider intersecting circles and ask whether triangles could be constructed by connecting the centres and intersection points of these circles. Furthermore, the same learning outcome expects students to reason about how to construct triangles with different side properties (equilateral, isosceles, scalene). As the process continued, the triangles drawn by the students varied depending on the positions of the circles they constructed. The following dialogue took place between the teacher and the students.

> S2: Teacher, mine turned out differently from yours, but why is that?
>
> T: At the beginning of the lesson, we talked about triangle examples from daily life. Aren't those triangles also different in size? For example, a triangular traffic sign and a house roof are not the same size, are they? So, children, even though the triangles we constructed on the board look different from each other, we've still drawn them correctly. And we'll soon learn together why they turned out differently.

Based on the above statements, it is expected that, in the later stages of the activity, students will be guided to understand why the triangles they constructed turned out differently. However, the following dialogues were observed as the activity progressed.

> T: Now, take out your compasses and rulers. Follow the steps on the board and apply them in your notebooks. While you're drawing in your notebooks, I'll show you the correct construction using GeoGebra.

Step 1: Attach your pencil to the compass, adjust the compass opening to 3 cm, and draw a circle.

Step 2: Do not change the compass width. Place the pencil tip at the centre of the circle you just drew and draw another circle. These two circles should pass through each other's centres.

Step 3: Connect the centres of the circles and one of their intersection points to draw a triangle. Do you think the side lengths of the triangle you have drawn are all equal, or are there any differences? What can you say about it? (Student responses are received.) Let's now measure the side lengths of the triangle we have constructed using a ruler. What did you notice when you measured the sides?

> S1: Teacher, all the sides are equal in length, they are all 3 cm.
>
> T: That's right, everyone. In this construction, we obtained a triangle in which all the sides are of equal length. Do you think this type of triangle has a special name? If so, what could it be?
>
> S2: A triangle with equal sides?
>
> S3: An equal-sided triangle?
>
> T: The triangle we have constructed has all sides equal in length. In mathematics, we call such triangles an equilateral triangle.

In the dialogue above, it is evident that the teacher provided students with a step-by-step instruction for constructing an equilateral triangle, yet did not include any inquiry into why the triangle was equilateral. At this stage, the teacher was expected to allow students to explore independently how triangles with different side lengths could be constructed using circles. The aim here was to guide students toward discovering the relationship between the size (radius) of the circles and the side lengths of the triangles. Therefore, Group 1 was considered to have demonstrated only a partially appropriate performance during the structured activity (exploration) phase, as they did not fully meet the pedagogical expectations of the task. In the later parts of the activity, the teacher applied similar procedures for constructing isosceles and scalene triangles.

In this phase of the activity, the prospective teachers stated that they employed the communication skill indicators of ‘using mathematical symbols and terms effectively and accurately’ and ‘applying mathematical language appropriately and effectively within mathematics itself, across different disciplines, and in daily life’. They also reported using the reasoning skill indicator of ‘making measurement estimations based on a specific reference point’. All of these indicators were deemed appropriate.

Explanation.(class.discussion).phase

In the relevant activity, it was observed that the teacher provided limited explanation aimed at helping students make sense of the processes carried out and did not adequately guide classroom discussions. During the structured activity, although students responded to a few questions posed by the teacher, these questions did not effectively direct students towards meaningful learning. Furthermore, one of the most notable shortcomings of the activity was the absence of inquiry into the why and how aspects of the learning process. Specifically, there was no exploration of why the constructed triangle was equilateral or how the size of the circles influenced the properties of the resulting triangle. Therefore, for the activity in question, due to the insufficient implementation of classroom discussions regarding the observed situations and the lack of adequate teacher-led explanations to support students in making sense of the process, the instructional approach has been categorised as partly appropriate in terms of alignment with constructivist teaching principles.

Findings derived from Group 5

The prospective teachers in Group 5 designed an activity based on the learning outcome “M.6.3.4.2. Constructs different rectangular prisms with a given volume using unit cubes and explains that volume is equal to the product of the base area and height with justification” (MoNE, 2018, p.63). The objective of the activity was expressed as ‘to help students realise that volume is equal to the product of the base area and height through the use of unit cubes’.

Engagement phase

T: Do you remember what we covered in the previous lesson?

S1: Teacher, we studied the volume of rectangular prisms.

T: So, how do we calculate volume?

S2: It’s the number of unit cubes that can fit inside, teacher.

T: That’s right, well done. Today, we’re going to do an activity. In this activity, each of you will act as a worker in a toy company. Each of you has a warehouse shaped like a rectangular

> prism, and you're responsible for it. Trucks are delivering boxes (unit cubes) to the company. The number of boxes you all receive is the same. You need to place these boxes into your warehouse without leaving any empty space. You will also need to record how many boxes you've placed along each edge of your warehouse in the provided table. Based on this information, go ahead and place the boxes into your warehouse space.

In this part of the activity, the prospective teachers stated that they utilised the communication skill indicator 'relating everyday language to mathematical language and symbols, and vice versa', as well as the connecting skill indicator 'connecting mathematics with topics and situations encountered in other subjects and daily life'. It was observed that mathematical terms such as 'rectangular prism', 'edge', and 'number of boxes' were used, indicating that the process involved a connection between mathematical and everyday language. Therefore, this indicator was coded as appropriate. Similarly, the connecting skill indicator was also coded as appropriate, as the situation involved the use of mathematical knowledge in a real-life context.

In the engagement phase of the activity, it was observed that the teacher provided students with information related to the lesson and reminded them of the learning outcomes from the previous lesson. Furthermore, the teacher's use of physical materials and the incorporation of a real-life scenario to initiate the activity were considered effective in capturing students' interest in the lesson. Therefore, this phase was coded as appropriate.

Structured activity (exploration) phase

At this stage, the given cubes are considered as boxes. Students begin placing the boxes into their warehouses. A certain amount of time is allocated, and the teacher moves around the classroom to observe what the students are doing and responds to any questions they may have. Once the time is up, a class discussion is initiated to allow students to make inferences.

> S1: Teacher, I've filled my warehouse. I used 32 boxes. There are no boxes left in the truck, so it means the volume of the truck is equal to the volume of my warehouse.

The prospective teachers stated that they used the indicator of the communication skill: 'Expressing mathematical thinking both orally and in written form.' This indicator was coded as appropriate.

> S2: Teacher, I've filled my warehouse too, and there are no boxes left in the truck, but the shape of the truck and my warehouse are not the same. How can their volumes be equal?
>
> S3: My warehouse has a different shape as well, and it's also different from yours, but it is completely filled.
>
> T: Your observations are excellent. So, what kind of conclusion can we draw from these observations?
>
> S1: Teacher, it seems that rectangular prisms can have the same volume even if they have different shapes.

The prospective teachers stated that they used the indicator of the reasoning skill as 'Making logical generalisations and inferences.' This indicator was coded as appropriate.

> T: Well done, children. As you have noticed, we can have rectangular prisms that have equal volumes but different shapes.
>
> S2: Teacher, in fact, the base of my prism turned out to be a square.

T: So, what do we call prisms that have square bases?

Class: Square prism!

T: That's right. Now, do you think a square prism can also be considered a rectangular prism?

S3: Teacher, since a square is a type of rectangle, I think we can also call a square prism a rectangular prism.

The prospective teachers stated that they utilised the reasoning skill indicator 'Making logical generalisations and inferences' as well as the communication skill indicator 'Expressing mathematical ideas through various forms of representation such as concrete models, shapes, images, graphs, tables, and symbols'. Both indicators were coded as appropriate.

Figure 4 The warehouses and their dimensions filled during Group 5's activity

Teacher: In the second phase of our activity, everyone will form groups of three and work collaboratively. Each group will no longer be responsible for a specific warehouse. Once again, you will receive boxes delivered by trucks. Based on the number of boxes you receive and ensuring that all boxes are used without leaving any empty spaces, each group will design three different rectangular prism-shaped warehouses. You will record in the given table how many boxes you placed along each edge of the warehouses you designed. Then, you will choose one person from your group, and that person will answer the questions.

The prospective teachers stated that they employed the communication skill indicator: 'Relating everyday language to mathematical language and symbols, and vice versa.' This indicator was considered appropriate.

T: Group 1, how many boxes did you use, and what were the edge lengths of the warehouses you constructed?

S1: Teacher, we used 27 boxes.

T: Group 2, how many boxes did you use, and what were the edge lengths of your warehouses?

S2: Teacher, we used 36 boxes.

T: Group 3, how many boxes did you use, and what were the edge lengths of your warehouses?

S3: Teacher, we used 48 boxes.

T: Now, let's display the tables where we wrote down the edge lengths.

	Segment 1	Segment 2	Segment 3
Fig 1	3	3	3
Fig 2	3	9	1
Fig 3	27	1	1

	Segment 1	Segment 2	Segment 3
Fig 1	6	6	1
Fig 2	3	6	2
Fig 3	3	3	4

	Segment 1	Segment 2	Segment 3
Fig 1	12	2	2
Fig 2	6	4	2
Fig 3	8	2	3

Figure 5 The second phase of Group 5's activity process

The prospective teachers stated that, in this part of the activity, they employed the communication skill indicator 'expressing mathematical ideas through various forms of representation such as concrete models, shapes, images, graphs, tables, and symbols' through the use of table representations. This indicator was considered appropriate.

> T: Well done, all three groups successfully built their warehouses. Now, what are the volumes of the warehouses you created, and how did you find them?
>
> S1: The volume of our warehouse is 27. We counted the number of boxes we placed inside.
>
> S2: The volume of our warehouse is 36. Teacher, we first built the base, then added layers of the same base on top. We used this method for all warehouses.

The prospective teachers indicated that, in this part of the activity, they employed the communication skill indicator 'expressing mathematical thinking both orally and in written form.' This indicator was considered appropriate.

> S3: The volume of our warehouse is 48. Teacher, we also counted the boxes. But it takes a lot of time, isn't there an easier way?
>
> S2: Counting one by one is a waste of time. We first built the base, then added layers on top.
>
> S1: But how did you decide how many boxes to put on the base?
>
> S 2: The first time, we placed ten boxes on the base, but when we added layers, they didn't form a proper prism. We tried again, and this time we used nine boxes for the base and managed to build exactly four layers. That's how we created a warehouse in the shape of a prism with a volume of 36. We used the same method to build our other warehouses. Teacher, we noticed something in the table.

The prospective teachers stated that, in this part of the activity, they made use of the communication skill indicator 'using mathematical symbols and terms effectively and accurately.' This indicator was considered appropriate.

> T: What did you notice?
>
> S2: When we looked at the values in the table, we saw that they all divide 36 exactly. So we realised that we needed to find the factors of 36 when constructing the warehouses.

The prospective teachers stated that, in this part of the activity, they made use of the reasoning skill indicator 'making logical generalisations and inferences.' This indicator was considered appropriate.

S 3: I think our friend is right, teacher. Our edge lengths also divide the volume exactly.

The prospective teachers also reported using the communication skill indicator ‘interpreting the accuracy and meaning of mathematical thinking.’ This indicator was considered appropriate.

T: Congratulations, everyone. Based on this information, what kind of conclusion can we draw?

S1: It means that if we multiply the three edge lengths we've written, we get the volume.

S2: We had first created the base, and we found the volume that way as well — by multiplying the base area by the other edge length.

The prospective teachers stated that they used the reasoning skill indicator ‘defending the validity and accuracy of inferences.’ This indicator was considered appropriate.

T: What is the name of that other edge? You had learned the elements of a rectangular prism, does anyone remember?

Class: Height, teacher!

T: That’s right. So how do we find the volume of a rectangular prism without counting the unit cubes one by one?

Class: Base area × Height, teacher!

The prospective teachers expressed that they used the connecting skill indicator ‘establishing relationships between concepts and operations.’ This indicator was considered appropriate.

Explanation.(class.discussion).phase

During this phase of the activity, it was observed that the teacher provided minimal direct explanations and instead guided the discovery process through question-and-answer interactions. Although the classroom discussion conducted during the discovery phase was considered appropriate, the fact that the teacher did not explicitly connect the steps of the process to the intended learning outcomes may hinder the activity from fully achieving its educational objectives. Therefore, this phase was coded as partly appropriate.

Findings derived from Group 7

The prospective teachers in Group 7 stated that the aim of the activity they designed, based on the learning objective “7.3.3.3 Calculates the area of a circle and a circular sector,” (MoNE, 2018, P.69) was ‘to enable students to discover the formula used to calculate the area of a circular sector.’

Engagement.phase

Teacher: Yusuf, the baker, is going to prepare a 40 cm diameter round flatbread for his son and 7 of his friends. Since each person will receive an equal slice of the flatbread, how can we calculate the central angle of each slice?

At this stage, the prospective teachers stated that the indicator of the connecting competency ‘connecting mathematics with topics and situations encountered in daily life’ was utilised. This situation was coded as appropriate.

It is observed that the engagement phase of the activity begins with a real-life problem. However, no additional steps were included to prepare students for the lesson or topic, to attract their attention, or to motivate them. Therefore, this phase was considered partly appropriate in terms of alignment with constructivist teaching.

Structured.activity.(exploration).phase

In the first stage of the exploration phase, students are guided to find the central angle of each slice of the circle that will be divided into equal parts. The dialogue between the teacher and the students during this stage is as follows.

> S1: Umm... well, since the diameter is 40 cm, maybe we can do something with that. Or... if we count the slices, maybe we can figure it out from there, but I'm not sure, teacher.
>
> T: That's a good start, what matters is that you're thinking. Let me give you a hint, does anyone remember how many degrees are in a full circle?
>
> S2: 360 degrees!
>
> S3: Then, if the flatbread is being divided among 8 people, we divide 360 by 8. So, 360 ÷ 8 = 45. That means each slice will have a central angle of 45 degrees.
>
> T: Now, let's find the area of the circle sectors shown here. (The teacher shows visuals of circles divided into 4, 6, and 8 equal parts on the board – Figure 6.) The first visual shows a circle divided into 4 equal slices. How can we calculate the area of one slice?
>
> S4: We can divide the total area by 4, teacher.
>
> T: And what about the second visual, where the circle is divided into 6 parts? What would we do here?
>
> S2: We divide the total area by 6.

Prospective teachers stated that they employed the reasoning skill indicator 'Making logical generalisations and inferences' during this process. This indicator was deemed appropriate for the given situation.

> T: Very good. So, how can we find the central angles of these slices?
>
> S1: We can measure them with a protractor.
>
> T: Then let's go ahead and measure them. (The students use protractors to measure.) What did you find?
>
> S: The first one is 90 degrees, the second one is 60 degrees!
>
> T: Great. Now, can we find them without using a protractor, just by calculation? For example, by dividing the total 360 degrees by the number of slices?
>
> S4: We divide 360 by the number of slices!
>
> T: Excellent! Now let's display this in the table. (The teacher guided the students to complete the table shown in Figure 6.)

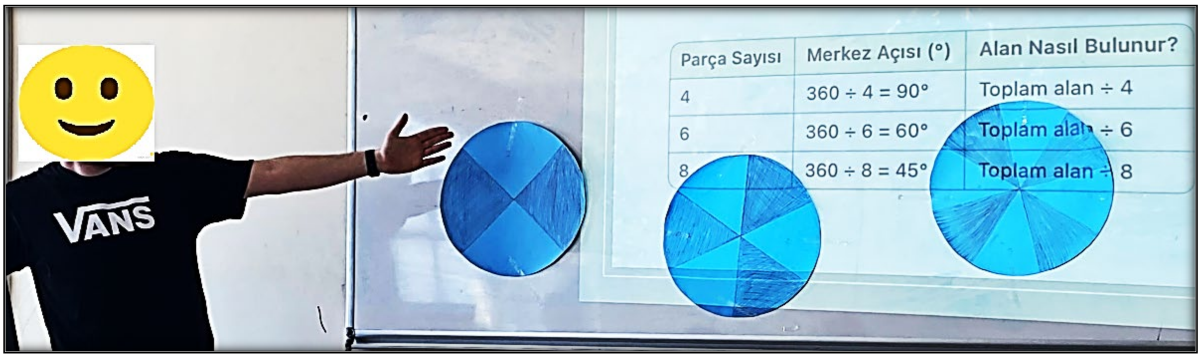

Figure 6 Exploration phase of Group 7 activity

At this stage, the prospective teachers stated that they made use of the communication skill indicator 'expressing mathematical ideas through various forms of representation such as concrete models, shapes, images, graphs, tables, and symbols'. This indicator was considered appropriate and was coded accordingly. The activity then continued with the following steps.

T: Look, children, what kind of relationship do you think there is between the central angle and the area?

S3: Both depend on the number of pieces, teacher!

T: That's absolutely right! So actually, we follow the same method for finding both the angle and the area, we divide the total 360 degrees by the number of pieces, and we also find the area of each equal slice by dividing the total area by the number of pieces.

In this section, it was stated that the indicator of the connecting skill, namely 'establishing relationships between concepts and operations', was used, and it was coded as appropriate.

T: Now let's go back to the flatbread example. We divided a flatbread with a diameter of 40 cm into eight slices, and each slice had a central angle of 45 degrees. So how do you think we can find the area of one of these slices?

S3: We can divide the total area by eight.

T: Correct! But think about it. How do we find the total area of the flatbread? Who remembers the area formula of a circle?

Students: $A = \pi r^2$.

T: Very good. If the diameter of the flatbread is 40 cm, what is the radius?

Students: 20 cm.

T: Then calculate the total area of the flatbread using $\pi = 3$.

Students: Area $= \pi r^2 = 3 \times 20 \times 20 = 1200$. If the total area of the flatbread is 1200, then dividing it by 8 gives the area of one slice: $1200 \div 8 = 150$.

The prospective teachers stated that the indicator 'using mathematical symbols and terms effectively and accurately' under the communication competency was employed in this part of the activity. This indicator was coded as appropriate.

T: So, could there be another way? Now pay attention. When we were calculating the central angles, we divided 360 degrees by the number of slices, right?

Students: Yes!

T: One slice had a central angle of 45 degrees. Now think about this. The total area of the whole circle is 400π and the full circle is 360 degrees. To find the area of just one slice, could we perhaps do this: total area × central angle of the slice ÷ 360? That is, 400π × 45 ÷ 360. Let's calculate and see if it works.

Students (after calculating): Yes, it works!

T: So, to find the area of a sector, we can say this. (The teacher writes the formula). That means we take the total area of the circle and proportion it to the central angle. You actually discovered this formula with your own reasoning!

Students: Wow! Yes!

T: Now you're not just students who know the formula for the area of a sector, you're students who discovered it!

It is evident from the activities carried out during Group 7's exploration phase that the prospective teachers did not fully grasp the true meaning of learning through discovery. During the process, the teacher initially demonstrated that both the central angle and the area of a circle sector are related to the number of equal parts. However, in the later stages of the activity, this knowledge was not utilized effectively. In the final stage, the teacher directly presented the formula to the students and had them verify its correctness through mathematical calculations. Therefore, this process was coded as not appropriate.

Explanation.(class.discussion).phase

Although class discussions were included in the activity developed by Group 7, it was observed that the teacher did not sufficiently clarify the connection between the activities conducted and the targeted discovery goals. Furthermore, the class discussions during the exploration phase were not adequately guided by the teacher. Therefore, this phase was coded as partly appropriate.

Findings derived from Group 10

The prospective teachers in Group 10 designed an activity aligned with the learning objective "M.6.3.5.2. Associates units of liquid measurement with units of volume measurement. Liquid measurement units are associated with volume measurement units to emphasise that liquid measures are essentially a specific type of volume measurement" (MoNE, 2018, p. 64).

The aim of the activity was described as to help students conceptualise that units of liquid measurement are a specific form of volume measurement by using measuring tools and concrete materials such as a graduated cylinder, cubic decimetres, rulers, and milk cartons when associating liquid and volume measurement units.

Engagement.phase

In the activity designed by Group 10, the teacher enters the classroom with a large water jug, several small water bottles, and a visual of a world map. On the board, the sentence 'Water Loss = A Danger to the Future?' is written. The teacher begins the lesson by asking the class: 'Children, today I want to show you something. This jug contains exactly 19 litres of water. Do you think this amount of water would be enough for a family for one day?'

In the engagement phase of the lesson, it is observed that materials, visuals, and questions that are likely to capture students' interest have been used. Therefore, this part of the process has been considered appropriate in terms of alignment with the constructivist approach.

Structured.activity.(exploration).phase

In the continuation of the activity, the following dialogue occurs between the teacher and the students.

> S1: Enough.
>
> S2: Not enough.
>
> S3: It depends on what it's being used for!
>
> T: Excellent thoughts! Now, let's make an estimate. How many litres of water do you think a person uses in a day?
>
> S1: 5 litres! Because we drink 2 litres a day and maybe use 3 litres for cleaning.
>
> S2: I think it's 30 litres because we're a family of six and each person drinks at least 2 litres. If we include water used for machines, cooking, and cleaning, it must be around that.

In this part of the activity, the prospective teachers stated that they used the reasoning skill indicator which involves 'making estimations about the result of operations and measurements by using strategies such as rounding, grouping suitable numbers, using the first or last digits, or their own developed strategies'. This indicator has been considered appropriate.

> T: In fact, an average person uses 135 litres of water per day. So not just this one large water bottle, nearly 7 of them! And that's just for one person!
>
> S1: But where do we even use that much water?
>
> T: Good question. Let's think together now! Since you woke up this morning, where have you used water?
>
> S1: Washing hands,
>
> S2: Brushing teeth.
>
> S3: Showering.

In this part of the process, the indicator of the connection skill 'Relating mathematics to topics and situations encountered in daily life' was reported to have been used. This indicator was considered appropriate and was coded accordingly.

> T: So, do you think people in Turkey or around the world always have access to this much water? Let's take a look at the map together. (The teacher opens the world map and points to regions experiencing water scarcity, parts of Africa, the Middle East, etc.) Look, in these areas, people sometimes have to walk for hours just to find one litre of water. Now, I want you to imagine you are in Africa. What kind of containers would you choose to carry water from the well back home? What would guide your choice of containers? Can we relate these containers to units of volume? Today, we will look for answers to these questions together. We'll explore how these units are used in daily life and also discuss how we can better conserve our water. (During the exploration phase, the teacher divides the class into groups and asks each group to follow the given instructions.)
>
> T: Today, we will think about water not just in litres, but also in terms of the space it occupies. Each group has 1 litre of water in front of them. You'll pour this water into different containers and find answers to some questions together. Now, one member from each group, please measure 1 litre of water with the measuring cylinder and pour it into the containers one by one.

Figure 7 Exploration process of Group 7

T: What did you observe?

S1: In some containers, the water looked deeper, in others it was more spread out.

T: So, what do you think that means?

S2: When the shape of the container changes, the appearance of the water changes, but the amount stays the same.

T: That's a great observation! So, does a litre only indicate the amount of water? Or is it also related to the space it occupies?

S3: I don't think it's related, teacher. For example, when my mom makes yogurt at home, she pours the milk from a 5L milk bottle into a pot to boil it. The pot fills up completely. So, the pot is also 5L, but the bottle and the pot have different shapes.

In this part, the prospective teachers stated that they used the indicator of the reasoning skill 'Relating mathematics to situations encountered in daily life.' This indicator has been considered appropriate.

T: (Shows a milk carton) Look at this container. Let's measure its edge lengths and think about how many liters of milk it might hold. Let's see how you approach this.

S4: Teacher, the base area of the milk carton is 10 cm × 5 cm = 50 cm^2. Since the height is 20 cm, if we place this base area on top of itself 20 times in centimeters, it would be 50 cm^2 × 20 cm = 1000 cm^3, which gives us the volume of the carton.

Here, the indicator of the reasoning skill 'Relating different mathematical concepts to one another' was reported to have been used, and since the concepts such as base area and height were related to the concept of volume, the use of this indicator was accepted as appropriate.

T: Excellent reasoning! So how would we express this in liters?

S2: Teacher, 1000 cm^3 = 1 L. Since 1 dm^3 = 1000 cm^3, can we say that 1 dm^3 = 1 L?

At this point, the indicators of the reasoning skill 'Making logical generalizations and inferences' and the communication skill 'Using mathematical symbols and terminology effectively and accurately' were reported to have been used. The use of both indicators was accepted as appropriate.

Students: Yes!

T: Then the unit of liquid measurement, the liter, also corresponds to a volume measurement. You made this discovery – congratulations!

When the process designed by Group 10 is examined, it is observed that students already had prior knowledge of the facts 1000 cm^3 = 1 L and 1000 cm^3 = 1 dm^3, and within the scope of this activity, they only arrived at the conclusion that 1 L = 1 dm^3. Here, it is seen that the prospective teachers interpreted the fact that students recognized the equivalence of 1 L = 1 dm^3 as a discovery that the unit of liquid measurement, the liter, also corresponds to a volume measurement. However, this situation is better regarded as a case of relating rather than discovering. Therefore, this part of the process was considered partly appropriate.

Explanation.(class.discussion).phase

In this phase, the teacher concludes the lesson with the following statements.

> T: Now, let's summarize together what we just discovered as a group. The amount of water didn't change, but it looked different in different containers. What was the reason for that?
>
> S1: Because the shapes of the containers were different. But the water always remained 1 liter, only its appearance changed.
>
> T: So, can we say how many cubic centimeters are in 1 liter of water in terms of volume?
>
> S2: I remember it's 1000 cm^3, because we calculated 5 cm × 10 cm × 20 cm.
>
> T: Yes! Then we can say the following: 1 L = 1000 cm^3. What do we notice through this conversion?
>
> S3: Liter might actually be the same as volume. It's just a special name used for liquids.

In this part, it was stated that the reasoning skill indicator 'making logical generalizations and inferences' was used, and it was coded as appropriate.

> T: That was a very well-articulated thought! We usually express liquid measurements in liters, but these units are not different from volume units. Therefore, we can say that 'liter' is actually a special unit of volume. (The teacher shows a 1-liter water bottle) This bottle holds 1 liter of water, which means 1000 cm^3. So, what is the benefit of this? Why would we want to relate liter to cm^3 in real life?
>
> S4: I think for calculation purposes. For example, if the volume of an aquarium is given in cm^3, we could figure out how many liters of water it can hold.

In this part, it was stated that the indicator of the connection skill 'Relating mathematics to situations encountered in daily life' was used, and it was accepted as appropriate.

> T: Yes, exactly. That was the main goal of our lesson today, to understand that units of liquid measurement are actually volume units, and to build a relationship between these two types of measurement.

In the explanation (class discussion) phase of the activity, it was observed that the emphasis was placed on the fact that 1 L = 1000 cm^3 and that a litre is a special unit of volume. However, it is understood that students had this knowledge prior to the activity. Therefore, it is evident that the outcomes of the discovery process were not aligned with the activities carried out during that phase. When the explanation phase is evaluated independently, it is considered appropriate; however, when evaluated in conjunction with the discovery phase, it has been coded as partly appropriate.

General findings obtained from the study

Within the scope of the study, the data regarding the pre-service teachers' activity design processes were analysed within the framework of pedagogical and skill dimensions, and the findings obtained are presented in detail in Table 1.

Table 1 General findings obtained from the study

Groups	Pedagogical Dimension			Skill Dimension								
				Connection			Communication			Reasoning		
	Eng.	Explor.	Explan.	A	PA	IA	A	PA	IA	A	PA	IA
G1	A	PA	PA	%100	-	-	%75	%25	-	%50	%50	-
G2	A	PA	PA	%100	-	-	-	%100	-	%25	%75	-
G3	A	PA	A	%100	-	-	%67	%33	-	%100	-	-
G4	A	A	IA	%100	-	-	%100	-	-	%78	%11	%11
G5	A	A	PA	%100	-	-	%83	%17	-	%100	-	-
G6	A	A	PA	%100	-	-	%100	-	-	%100	-	-
G7	PA	IA	PA	%100	-	-	%100	-	-	%100	-	-
G8	PA	PA	A	%100	-	-	%67	%33	-	%75	%25	-
G9	NA	PA	A	%100	-	-	%100	-	-	%50	%33	%17
G10	A	PA	PA	%100	-	-	%100	-	-	%100	-	-
G11	PA	PA	A	%100	-	-	%87	%13	-	%100	-	-
Mean	A	PA	PA	%100	-	-	%80	%20	-	%80	%18	%2

A: Appropriate, PA: Partly appropriate, IA: Inappropriate, Eng: Engagement, Explo.: Exploration, Explan.: Explanation

Considering the pedagogical dimension, it can be stated that the prospective teachers generally demonstrated appropriate performance in the engagement phase, while showing partly appropriate performance in the exploration and explanation phases of their instructional activity designs. The engagement phase appears to be the most successful, as only one group was evaluated as 'not appropriate', indicating that an overall sufficient level of performance was achieved in this dimension. The exploration phase stands out as the most challenging phase for the prospective teachers, since only three groups were able to demonstrate an appropriate level of performance. In the explanation phase, four groups were coded as 'appropriate', six as 'partly appropriate', and one as 'not appropriate'. This indicates that prospective teachers performed at varying levels in terms of including explanations that support conceptual understanding.

When the data obtained from the skill dimension are examined, it is seen that the prospective teachers generally demonstrated appropriate performance across all process skills. However, when the skills are compared with one another, it can be said that the best performance was observed in the connection skill, followed by communication and reasoning skills, respectively. In other words, prospective teachers showed the lowest performance in the reasoning skill. When the data related to process skills are examined in more detail, the findings presented in Table 2 were obtained.

Table 2 Findings obtained from the indicators related to process skills

MS	Relevant indicator	A	f	PA	f	IA	f	Tf
Connection	Connecting mathematics with topics and situations encountered in other subjects and daily life	G1 (1), G2 (1), G3 (1), G4 (1), G5 (1), G6 (1), G7 (1), G8 (1), G9 (1), G10 (3), G11(2)	14	-	-	-	-	14
	Establishing relationships between concepts and operations	G3 (4), G4 (2), G5 (1), G7 (1), G11(2)	10	-	-	-	-	10

	Connecting various mathematical concepts to one another	G4 (1), G5 (1), G6 (1), G10 (1), G11(3)	7	-	-	-	-	7
	Representing mathematical concepts and rules using different forms of representation	G8 (1), G9(1)	2	-	-	-	-	2
	Relating everyday language to mathematical language and symbols, and vice versa	G3 (1), G5 (1), G11(1)	3	G1 (1), G5 (1)	2	-	-	5
	Expressing mathematical ideas through various forms of representation, such as concrete models, shapes, images, graphs, tables, and symbols	G1 (1), G4 (3), G5 (2), G6 (1), G7 (1), G8 (1), G9 (2), G11(1), G3(1)	13	G2 (1), G3 (2), G8 (1)	4	-	-	17
Communication	Using mathematical symbols and terms effectively and accurately	G1 (1), G3 (2), G5 (1), G6 (1), G7 (1), G8 (3), G9 (1), G10 (1)	11	G3 (3)	3	-	-	14
	Applying mathematical language appropriately and effectively within mathematics itself, across different disciplines, and in daily life	G1 (1), G11(2)	3	G3 (1)	1	-	-	4
	Expressing mathematical thinking both orally and in written form	G3 (6), G4 (5), G5 (2), G6 (1), G8 (2), G9 (4), G11(7)	27	G3 (1)	1	-	-	28
	Interpreting the accuracy and meaning of mathematical thinking	G3 (4), G5 (1), G6 (1), G9 (1), G11(3)	10	G8 (1), G11(2)	3	-	-	13
	Recognising that mathematics is a language with its own unique symbols and terminology;	-	-	G8 (1)	1	-	-	1
	Making estimations about the results of operations and measurements using strategies such as rounding, grouping appropriate numbers, or focusing on the leading or trailing digits, as well as self-developed strategies	G10 (1)	1	G1(1)	1	G4 (1), G9 (1)	2	4
	Making measurement estimations based on a specific reference point	G1 (1)	1	G2 (1)	1	-	-	2
Reasoning	Making logical generalisations and inferences	G2 (1), G3 (5), G4 (1), G5 (3), G6 (3), G7 (2), G8 (1), G9 (2), G10 (2), G11(7)	27	G2 (1), G8 (1)	2	-	-	29
	Explaining and using mathematical patterns and relationships when analysing a mathematical situation	G4 (3), G9 (1)	4	-	-	-	-	4
	Defending the validity and accuracy of inferences	G4 (3), G5 (1), G6 (1), G8 (2)	7	G4 (1), G9 (2) G2(1)	4	-	-	11

MS: Mathematical skill, A: Appropriate, PA: Partly appropriate, IA: Inappropriate, f: Frequency, Tf: Total frequency

When the data presented in Table 2 are examined, it is observed that all indicators of the skill of making connections were used appropriately at a rate of 100%. In addition, regarding the communication skill, the three most successfully utilised indicators were, respectively, 'Expressing mathematical thinking both orally and in written form' (96%), 'Using mathematical symbols and terminology effectively and accurately' (79%), and 'Interpreting the accuracy and meaning of mathematical thinking' (77%). As for the reasoning skill, the most successfully demonstrated indicators were 'Explaining and using mathematical patterns and relationships when analysing a mathematical situation' (100%), 'Making logical generalisations and inferences' (93%), and 'Defending the validity and accuracy of inferences' (64%). On the other hand, the indicator 'Making estimations about the result of operations and measurements using strategies such as rounding, grouping suitable numbers, using the first or last digits, or their own developed strategies' had the lowest success rate, with only 25%. The success rate for the indicator 'Making measurement estimations based on a specific reference point' was found to be 50%.

Discussion

The findings of this study, which examined mathematical activities designed by elementary mathematics prospective teachers within the framework of a case study and supported by mathematical process skills, revealed that prospective teachers demonstrated a higher level of success in the skill dimension compared to the pedagogical dimension. The prospective teachers adequately addressed the majority of the indicators related to mathematical process skills. This suggests that the participants were able to comprehend the meaning of the relevant indicators and reached a sufficient level of reflection in incorporating these skills into their instructional designs. In other words, the participants were generally successful in identifying the mathematical content and structuring it within the framework of process skills. However, in the pedagogical dimension, particularly in processes related to the structural planning of the lesson, notable deficiencies were observed. Specifically, it was found that the prospective teachers had difficulties in the "exploration" and "explanation" phases of the instructional process they designed. In the exploration phase, deficiencies were identified in creating guiding questions and instructions that would enable students to construct knowledge, as well as in organising the process in a student-centred manner. Similarly, in the explanation phase, participants often provided inadequate, structured explanations that hindered students' ability to make sense of their discoveries and transform them into conceptual understanding. Moreover, these explanations were frequently not presented consistently within the context of the activity and lacked the structure necessary to support students' cognitive processes.

Upon examining the relevant literature, it is possible to encounter similar findings. In their study on the processes through which prospective teachers develop mathematical activities, Canbazoğlu and Tarım (2021) reported that prospective teachers preferred approaches such as cooperative learning, discovery learning, and the constructivist approach. However, they also stated that these teachers faced difficulties during the design process of the activity. Çakmak-Gürel (2023) found that while prospective teachers were able to develop activities for out-of-school learning environments, these activities were limited in terms of instructional techniques. Similarly, Çenberci and Özgen (2021) noted that prospective teachers encountered specific challenges in designing mathematics activities that incorporate real-life contexts. In their study investigating prospective teachers' perceptions regarding the design of mathematics learning activities, Toprak et al. (2017) noted that the designed activities lacked essential features expected in effective instructional practices. Özgen and Alkan (2014) observed that many of the activities developed by prospective mathematics teachers did not facilitate students' active cognitive and physical participation, instead reflecting teacher-centred processes. Likewise, Özgen (2019) identified several challenges that prospective mathematics teachers encounter when designing learning activities. The literature also shows that not only prospective teachers, but also in-service mathematics teachers experience difficulties in designing instructional activities (Bal, 2008; Bozkurt, 2012; Bozkurt & Kuran, 2016; Sağıroğlu & Karataş, 2018; Uğurel et al., 2010). Therefore, these studies suggest that while activity design is considered beneficial for learning, it is also perceived as a challenging task in terms of implementation (Séré & Beney, 1997).

One of the significant findings of the study is that the exploration phase was the most challenging stage for the prospective teachers during the instructional activity design process. This result suggests that participants struggled to create student-centred learning environments, a core principle of the constructivist learning approach. Since the exploration phase requires the teacher to facilitate students' active engagement in constructing knowledge, it was observed that prospective teachers struggled particularly with designing guiding but non-directive strategies in this stage. In contrast, their performance in the explanation phase was found to be partly appropriate. Although prospective teachers attempted to define and structure mathematical concepts during this phase, their explanations often remained superficial and lacked depth. Moreover, it was frequently observed that the instruction in this phase was conducted in a teacher-centred manner. The relevant literature presents consistent findings. For instance, Hacısalihoğlu-Karadeniz (2019) noted, in a study examining prospective mathematics teachers' use of the discovery-based learning approach, that while they possessed theoretical knowledge about the approach, they required further support in its practical application. Similarly, Yılmaz and Ev-Çimen (2011) reported in their research on the design and implementation of lesson plans aligned with the 5E model that prospective teachers exhibited insufficient performance in both the exploration and explanation stages. Biber et al. (2015) found that 64% of teachers implementing the 5E model experienced challenges, particularly in the exploration stage. In another study, Turan (2021) emphasised that prospective teachers found it challenging to shift toward a more student-centred teaching model during the exploration and explanation stages of the 5E instructional model. Taken together, these findings from the literature reinforce the conclusion that designing the exploration and explanation phases remains a pedagogical challenge for prospective teachers, especially in maintaining a constructivist and student-centred orientation throughout instructional activities.

When the findings obtained regarding the skills dimension of the study are examined, it is evident that the prospective teachers demonstrated particularly low levels of performance in identifying and applying indicators related to estimation-based reasoning skills. A review of the literature reveals similar findings across various studies. For example, Jones (2000), Harel (2001), De Castro (2004), and Moralı et al. (2006) all draw attention to the inadequacy of prospective teachers' reasoning skills. One of the most striking findings of the present research is that only 25% of the participants successfully demonstrated the skill of 'making estimations about the result of operations and measurements using their own strategies'. Similarly, only 50% were able to perform estimations based on a given reference point, suggesting that the prospective teachers had not fully internalised the relevant indicators of reasoning skills. Comparable results have been reported in other studies. For instance, Kılıçoğlu and Özdemir-Baki (2022), in their research on classroom teacher candidates' perceptions of mathematical process skills, found that participants did not mention any estimation strategies. Bozkurt and Yavaşça (2021) revealed that while teacher candidates valued estimation skills, they held incomplete conceptions of what those skills entailed. Özcan (2015) similarly highlighted a lack of proficiency and awareness among both primary mathematics and classroom teachers regarding estimation skills. Towers and Hunter (2010) specifically described estimation in measurement as a complex domain that requires integrating the concepts of measurement and estimation through logical reasoning processes. This underscores that the use of such skills in pedagogical contexts is closely tied to both content knowledge and reasoning ability. Although there is limited research focusing specifically on this area, Subramaniam (2014) found that while prospective teachers possessed specific benchmarks for estimating length, these were not clearly reflected in their pedagogical content knowledge. In their 2024 study, López-Serentill investigated prospective teachers' estimation strategies and reported that while more than 80% of participants could produce reasonable estimates and apply strategies for length, they faced significant difficulties with area, volume, and capacity estimation, often relying heavily on formula-based reasoning. This reliance suggests that the participants were unable to make estimations using appropriate referential reasoning. These findings are consistent with recent research indicating that prospective teachers frequently struggle with concepts of area and volume, relying on procedural or formula-based understandings (Runnalls & Hong, 2020; Seah & Horne, 2020). In this context, Çilingir Altıner (2024) emphasised that integrating real-life contexts and practical applications into mathematics education can help prospective teachers develop more accurate and meaningful estimation strategies. Similarly, Türk and Ev-Çimen (2022) and Tekinkır (2008) stressed the importance of designing learning activities that support the use of diverse estimation strategies.

Limitations and future directions

This study was conducted with 55 prospective mathematics teachers and examined their activity design processes across two main dimensions: the pedagogical dimension and the skills dimension. The findings provide valuable insights into prospective teachers' abilities to plan and implement instructional activities based on the constructivist learning approach. However, the research was limited to a relatively small sample group enrolled in a single teacher education program, which restricts the generalizability of the results. Future studies may benefit from including larger and more diverse sample groups from different institutions to allow for comparative analysis of the findings. The data collection process of this study was limited to a single academic term, and data were obtained through activity reports, observation notes, and group interviews. While these methods provided in-depth qualitative data, longitudinal or mixed-method research designs are recommended to better monitor the development of prospective teachers' planning and pedagogical decision-making skills over time. Additionally, the participants' limited experience in designing activities based on constructivist models may have hindered their ability to develop sufficiently detailed instructional strategies, particularly in the exploration and explanation phases. Therefore, future research could adopt a process-oriented approach in which prospective teachers are provided with structured feedback and scaffolded guidance, enabling closer monitoring and support for the development of their pedagogical design competencies.

Conclusion

The findings of this study indicate that prospective teachers were generally successful in integrating mathematical process skills, such as reasoning, communication, and connections, into their instructional activities. However, they appeared to need more support in designing structurally sound lesson plans from a pedagogical perspective. In this regard, it is recommended that teacher education programs place greater emphasis on practice-based, structured, and reflective activities aligned with constructivist teaching models. Providing prospective teachers with opportunities to experience constructivist learning environments and deepening their understanding of these environments is critical for enhancing their instructional design skills and promoting student-centred teaching approaches. Activities such as receiving feedback on designed tasks, engaging in peer collaboration, and revisiting the design process can help prospective teachers transform their content knowledge into effective pedagogical practices. In line with this, Turan and Matteson (2020) state that although prospective teachers are introduced to constructivist environments during instructional methods courses at the university, they often struggle to apply this knowledge in practice due to a lack of sufficient hands-on experience. Similarly, Enugu and Hokayem (2017) argue that offering prospective teachers opportunities to translate theory into practice and derive theory from practice can help close the gap between theoretical knowledge and its practical application.

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