2025, Vol 4, No 1, 79-92
https://doi.org/10.29329/pedper.2025.100

Pedagogical Perspective

RESEARCH ARTICLE Open Access

# Exploring the creative pathways: How a second-grader navigates multiplication and division without formal instruction

Saniye Nur Ergan*[1]

[2]Department of Primary Education, Faculty of Education, Ordu University, Ordu, Türkiye.

**Abstract**

This study explores how young learners develop self-invented strategies for multiplication and division without formal instruction, aiming to investigate a second-grade student's problem-solving methods, reasoning processes, and challenges. A qualitative case study design was followed, involving a student with no prior formal instruction in multiplication or division. Data collection included a series of problem-solving sessions, where the student tackled tasks using her strategies. Observational notes, student drawings, and verbal explanations were analyzed qualitatively to identify patterns and challenges in her approach. The student revealed a range of self-invented strategies, including repeated addition, visual representations, and intuitive reasoning. She progressed from concrete methods to more abstract thinking, successfully solving many problems. However, challenges were noted, including confusion with the commutative property and difficulty with more complex division tasks. Despite limited formal instruction, her ability to articulate and apply mathematical concepts underscored the value of prior informal learning experiences. The findings highlight the importance of recognizing and supporting self-constructed strategies in early mathematics education. By leveraging students' intuitive methods and addressing conceptual gaps, educators can enhance mathematical understanding. Future research should investigate how informal learning experiences contribute to early mathematical development and explore instructional approaches that integrate these self-invented strategies.

**Keywords:** Informal learning, Visual tools, Early math, Problem solving.

## Introduction

Mathematics education has followed a structured progression, where students are introduced to gradually more complex concepts through formal instruction. However, growing research highlights the inherent problem-solving abilities of young children, often demonstrated before formal teaching begins (Clements & Sarama, 2020). This growing recognition of children's natural problem-solving skills has led to a shift in perspective, prompting increased interest in exploring how students construct mathematical knowledge independently. Young learners often use intuitive methods, including repeated addition, grouping, and visual representations, to solve mathematical problems. These self-invented strategies are vital to developing mathematical reasoning, reflecting a deeper understanding of concepts that may not yet be formally taught.

*Corresponding Author: snurergan@gmail.com

Received 02.01.2025 Revised 11.02.2025 Accepted 19.02.2025

Pedagogical Perspective

Boaler (2024) emphasizes that allowing students to explore mathematics through their own strategies fosters both a greater sense of ownership and deeper conceptual understanding. Van de Walle et al. (2016) also argue that when supported throughout the learning process, self-invented strategies can enrich students' mathematical reasoning. In line with this perspective, the current study examines how a second-grade student, who has not received formal instruction in multiplication and division, navigates and solves problems using her self-developed strategies.

Despite the growing recognition of students' ability to construct their own mathematical strategies, research on how these strategies evolve, particularly in the context of multiplication and division, remains largely unexplored. Recent research has highlighted gaps in our knowledge about how young learners mathematical reasoning (Jonsson et al., 2022). This study seeks to address these gaps by exploring how a second-grade student, without formal instruction, constructs and applies problem-solving strategies in multiplication and division, while also shedding light on the changes of her mathematical reasoning throughout the process. Multiplication and division were chosen as focal points of this study due to their foundational role in mathematical understanding. Research indicates that an early conceptual understanding of these operations is crucial, as they serve as building blocks for proportional reasoning, algebraic thinking, and problem-solving skills (Zager, 2023). Furthermore, multiplication and division require students to extend their reasoning beyond basic counting strategies, promoting a shift toward relational thinking and structured problem-solving approaches (Hulbert et al., 2023). The development of problem-solving skills during the early elementary years is particularly crucial, as this period represents a critical window for mathematical growth. Research by van Oers (2024) highlights that when young children are provided with appropriate support and the opportunity to solve problems independently, meaningful learning is fostered. In this sense, the early elementary years serve as a foundation for students' mathematical identity and self-efficacy, making it essential for children to develop confidence in their ability to approach and solve novel problems.

The following questions guide the research:

1. What self-invented strategies does a second-grade student use to solve multiplication and division problems without prior formal instruction?
2. How do visual and concrete representations support the student's problem-solving process?
3. What challenges and misconceptions arise as the student constructs her own strategies, and how can these inform instructional practices?

This investigation is grounded in constructivist learning theory, which posits that learners actively construct knowledge through exploration and problem-solving. According to this perspective, self-invented strategies are not merely steps toward formal methods but are legitimate forms of understanding. Constructivism highlights the value of the learning process itself, with a focus on how students actively make sense of the world through their own experiences and reflections. The findings from this study can potentially inform instructional practices in early mathematics education by highlighting the importance of self-invented strategies. Traditional approaches, which often focus on procedural knowledge and memorization, can limit conceptual understanding and creative exploration opportunities. This research emphasizes the need to create an environment where young learners' intuitive strategies are recognized and supported, allowing for a deeper, more meaningful understanding of mathematical concepts. Additionally, by identifying challenges and misconceptions that arise in students' self-invented strategies, this study offers insights into how instruction can better address these issues while fostering creativity and exploration in mathematical learning.

## Method

### Design

This study employs a qualitative approach to explore the self-invented strategies of a second-grade student in solving multiplication and division problems without prior formal instruction. The qualitative method was chosen for its ability to provide rich, detailed insights into the participant's mathematical thinking processes and problem-solving strategies (Cyr & Goodman, 2024).

### Participants

The student was purposefully selected based on her academic readiness, engagement with mathematical reasoning, and ability to express her thought processes. Additionally, the researcher had previously collaborated with the student on another project, which created a familiar and supportive environment that helped alleviate any potential anxiety and foster self-expression. Ethical considerations were carefully followed, with parental consent obtained and the participant's voluntary participation ensured. The study took place in a quiet, familiar classroom, chosen to minimize distractions. Three semi-structured sessions, each lasting 50-60 minutes, were held, with the goal of gradually introducing multiplication and division tasks. These sessions involved problem-solving activities set in real-world contexts, along with visual representations and hands-on materials, to explore the student's reasoning. The flexible, inquiry-driven approach allowed the researcher to adjust the questions according to the student's responses, facilitating a deeper exploration of her self- invented strategies.

### Data collection

Data were collected through multiple methods to ensure a rich and comprehensive understanding of the student's problem-solving strategies. The process involved three sessions, during which problem-solving tasks, written work samples, and non-structured observation notes were collected. These data collection tools were carefully designed to capture different aspects of the participant's reasoning processes. The problem-solving tasks were created using Van de Walle's (2016) framework of problem types, which was adapted to the student's grade level and prior mathematical knowledge. These tasks progressed from simple repeated addition problems to more complex division scenarios (Haylock & Manning, 2019). The written work samples consisted of the student's written responses to these tasks, which were analyzed for patterns in reasoning and strategy development. Non-structured observation notes were taken throughout the sessions, documenting the student's explanations, challenges encountered, and any strategies used. This diversification of methods, including written and observational data, ensured a more holistic view of the participant's mathematical reasoning, enhancing the reliability and validity of the findings.

To ensure the validity and reliability of the data collection tools, the problem-solving tasks and written work samples were reviewed by two mathematics education experts who provided feedback on their alignment with developmental appropriateness and educational standards.

Expert opinions helped refine the tasks to ensure they were relevant and accurately assessed the participant's problem-solving readiness. Reliability was ensured by following consistent data collection protocols across all sessions and using a systematic approach to coding and analyzing the written responses and observation notes (Greig et al., 2013).

The sessions included a series of progressively challenging tasks that were sequenced to gradually increase in complexity, starting with basic repeated addition and advancing to more challenging division problems. During the sessions, the student was encouraged to verbalize her thought process and use visual or concrete tools (e.g., drawings and manipulatives) to solve the problems. In the initial phase of the study, it was necessary to assess whether the student encountered difficulties with addition and subtraction. To achieve this, 11 routine mathematics problems were designed based on the varied problem structures outlined in Haylock and Manning's (2019) framework. The problems incorporated two fictional characters, Aslı and Selin, to maintain the student's engagement during the activity, as Van de Walle et al. (2016) recommended. During a 55-minute session focused on addition and subtraction, the student demonstrated no difficulties solving, explaining, or visualizing the problems. She effectively utilized algebraic representations appropriate for her grade level, including symbols such as '?', '+,' '-,' and '=.' Additionally, the student actively participated in generating her own addition and subtraction problems. A review of her notebook, combined with insights from discussions with her classroom teacher and observations during the problem-solving session, confirmed that she was capable of solving and explaining a variety of addition and subtraction problems. These findings indicated that the student was ready to progress to multiplication and division tasks.

## Procedure and researcher's role

In this study, the researcher's role was to observe, document, and facilitate the participant's development of problem-solving strategies. Guided by a constructivist approach, the researcher aimed to create an environment where the student could actively engage with tasks, reflect on her thinking, and generate her own solutions. Prior experience with the student, established through a previous project, allowed for a more flexible and self-expressive collaboration. This familiarity with her learning style helped tailor tasks to her existing knowledge and cognitive abilities, ensuring the introduction of multiplication and division tasks in a gradual, manageable way. The researcher's role was to facilitate and observe how the student's strategies evolved over time, using non-structured observations to gain deeper insights into her reasoning. While past interactions shaped the researcher's understanding of the student, this insider perspective enriched the study. However, it also introduced subjectivity into the interpretation of the data, a common feature of qualitative research. This relational dynamic between the researcher and participant influenced the design of the study and its outcomes, providing a rich, nuanced understanding of the student's learning process. The study progressed through several sessions, each designed to assess and develop the participant's problem-solving strategies.

Session 1: The first session focused on assessing the participant's readiness for multiplication and division tasks by evaluating her proficiency in addition and subtraction. This initial assessment helped determine whether she had a sufficient foundation in basic arithmetic operations to proceed with more complex tasks.

Sessions 2 and Beyond: In the subsequent sessions, the participant was presented with problems that required multiplication or division reasoning. These problems were designed to encourage her to think critically and apply her existing knowledge to solve them. Throughout these sessions, she was prompted to verbalize her thought process, which allowed insight into her reasoning and allowed her to refine her strategies. Additionally, visual representations, such as drawings and manipulatives, were encouraged as tools to support problem-solving and help the participant visualize the relationship between numbers.

### Data analysis

The data were analyzed using content analysis, a method that systematically examines text to identify recurring patterns, themes, and categories. This analysis was performed using qualitative analysis software, ensuring consistency and rigor throughout the coding process. To ensure reliability and validity, a second expert in qualitative research was consulted during the coding process. This expert provided feedback on the analysis of the data, helping to refine the analysis and confirm that the emerging themes accurately reflected the participant's reasoning strategies. Both the written work and observation notes were coded to uncover recurring themes, which were organized into broader categories, such as the use of visual representations, repeated addition, and intuitive division strategies. The content analysis aimed to understand how the student developed and applied her methods, exploring how these strategies evolved over the course of the sessions and highlighting the progression and refinement of her problem-solving approaches.

## Findings

This study explores the inventive strategies employed by a second-grade student to solve multiplication and division problems without prior formal instruction. The focus is on how the student, unaided by explicit teaching, constructs her own methods to navigate these mathematical challenges. The findings reveal a rich array of self-invented strategies characterized by creativity, adaptability, and a reliance on visual and concrete representations. Below, the findings are organized thematically, with each theme highlighting the student's unique problem-solving approaches and accompanied by interpretive commentary.

### From repeated addition to pattern recognition

The student's initial approach to multiplication problems was heavily grounded in repeated addition, a strategy she intuitively adopted to understand multiplicative relationships. For example, when asked, "Aslı has two 10 TL bills. How much money does she have in total?" the student initially responded with "*10*," but quickly corrected herself, reasoning, "*Since there are two 10 TL bills, adding 10 and 10 gives 20. It's very easy*." This response highlighted her use of repeated addition as a foundational strategy for multiplication. As the student encountered more complex problems, she started recognizing underlying patterns in the numbers. This shift from a purely additive approach toward a more pattern-based strategy marked the next stage in her mathematical development. For instance, when calculating the total for six 10 TL bills, she counted by tens: "10, 20, 30, 40, 50, 60," and concluded, "*I could just say 60 without counting*." Here, the student demonstrated the ability to recognize a multiplication pattern based on the number 10, which enabled her to skip the addition steps and arrive at the solution more efficiently.

Her ability to detect and use patterns became even more apparent when she encountered problems with different sets of numbers. For example, when asked to calculate the total for three plates, with five candies on each plate, she added: "5 + 5 + 5," resulting in 15. The same recognition of patterns appeared in later problems, such as when she was tasked with finding the total for two plates with seven candies each. Instead of relying on basic counting, she quickly calculated 7 + 7 = 14, demonstrating her recognition of the pattern that multiplication is essentially the repeated addition of the same number. The student's development of pattern recognition was not just limited to additive strategies. For example, in a problem asking for a total of three 10 TL bills, she immediately answered with "30" and wrote her solution as 20 + 10. When prompted to explain her thinking, she mentioned, "*I didn't want to write 10 three times, so I used 20 instead*." In doing so, she leveraged her emerging understanding of grouping to recognize that adding 10 three times could be simplified by using "20+10".

The most significant shift occurred when the student, after solving multiple problems, began to identify multiplication patterns as a systematic approach rather than relying on random calculations. When asked to solve "10 tens," she initially hesitated, but after observing the relationship between groups of ten, she wrote out the sum of 10 tens through rhythmic counting. She then noted, "*The shortest way is this*," marking the moment she fully embraced pattern recognition in multiplication. The student realized that by recognizing the pattern in the grouping of tens, she could multiply quickly without having to count each number one by one. This shift illustrates a clear progression in her understanding from reliance on repeated addition to recognizing multiplication as the sum of equal groups—a core principle of multiplication. As her problems became more complex, she continued to rely on patterns, such as grouping similar numbers, which facilitated faster calculations. Overall, these findings suggest that the student began developing pattern recognition as a core skill in multiplication. Initially using repeated addition, she later identified patterns in number relationships that allowed her to simplify and accelerate her calculations. This transition from concrete addition to recognizing and using patterns is a crucial development in building her multiplicative reasoning, setting the stage for more abstract and efficient mathematical strategies.

## Drawing and manipulatives as tools for understanding

The student frequently relied on visual and concrete representations to model and solve problems, enhancing her understanding of mathematical concepts. For example, when asked, "Aslı has 4 bags of apples, and each bag contains 6 apples. How many apples does Aslı have in total?" she drew four bags, each containing six apples, and counted the total to arrive at 24. This use of drawings allowed her to visualize the problem, which helped her ensure the correctness of her solution through a tangible representation (Fig. 1).

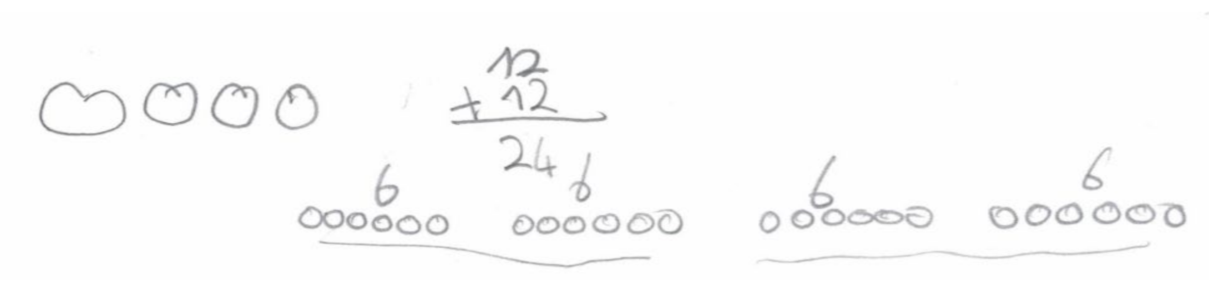


**Figure 1**

Similarly, in a problem involving grouping, "Aslı has 24 apples and wants to pack them into bags that can hold 6 apples each. How many bags will she need?" the student drew 24 apples and grouped them into six sets. She then counted the number of groups and arrived at the correct answer, 4. This self-invented grouping strategy illustrates her ability to use concrete methods (Fig. 2) to solve division problems, even without formal instruction.

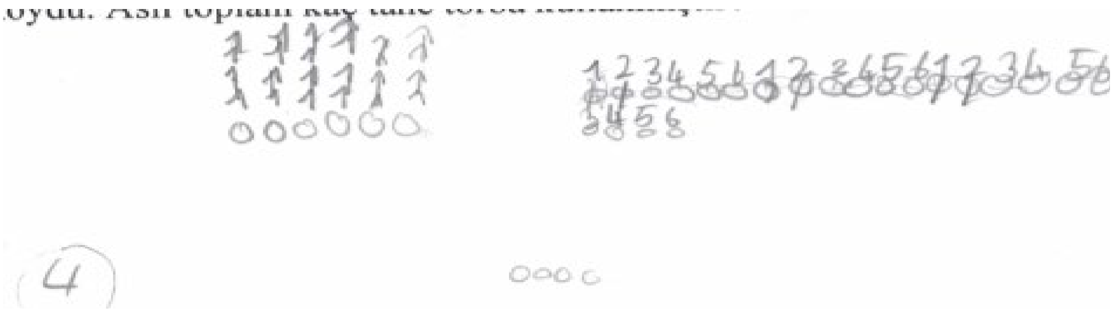


Figure 2

The student also used manipulatives, such as chickpeas, to model problems. For instance, when asked to solve 3 groups of 5, she physically grouped five sets of three chickpeas. This hands-on approach allowed her to explore the problem in a concrete way, reinforcing her understanding of multiplication as grouping. Her use of manipulatives highlights the importance of tangible tools in helping young learners construct their own strategies for solving mathematical problems. These findings underscore the critical role of visual and concrete representations in the student's problem-solving process. Her reliance on drawings and manipulatives enabled her to model and understand problems and provided a method to verify solutions. By incorporating these tools, the student was able to make abstract concepts more accessible and manageable, demonstrating the value of using visual aids in early mathematics education. This approach fosters a deeper understanding of mathematical principles by offering students concrete methods to engage with and solve problems, particularly when working with abstract ideas.

## Emerging understanding of the commutative property

The student demonstrated an intuitive grasp of the commutative property of multiplication, although this concept was not explicitly taught. For instance, when solving 3 groups of 5, she initially added 5 + 5 + 5, but later recognized that 5 groups of 3 would yield the same result. This insight, though intuitive, was not always consistent. When the order of factors was reversed, she occasionally struggled to visualize the difference between "3 groups of 5" and "5 groups of 3." Her initial confusion between the two expressions revealed a developing understanding of the commutative property. This partial understanding was evident in her difficulty applying the commutative property in all cases. Despite her ability to explore and discover mathematical relationships independently, the inconsistencies in her reasoning suggested that her grasp of the concept was still in the process of solidifying.

For example, when attempting to visualize 3x5 and 5x3, she drew a representation (Fig. 3) to aid her thinking, as shown in the figure above. While drawing the groups clarified the problem’s structure, she faced challenges applying the commutative property across different scenarios.


Figure 3

The student’s partial grasp of the commutative property highlights the importance of reinforcing concepts and providing explicit guidance to address misconceptions and deepen understanding. Interestingly, when asked to represent the expressions 3×5 and 5×3 without being given a problem context, the student appeared confused. However, it was observed that she had no difficulty visualizing such multiplication operations when presented in the form of word problems (see Fig. 6). This finding emphasizes the role of contextual word problems and their components in fostering abstract thinking in mathematics education.

## Intuitive division strategies: Equal sharing and grouping

The student's approach to division problems was characterized by intuitive strategies such as equal sharing and grouping. For example, when asked to divide 24 apples among three friends, she drew 24 apples and distributed them individually to each friend, eventually recognizing that each friend would receive eight apples. This self-invented equal-sharing strategy allowed her to conceptualize division as a process of fair distribution (Fig. 4).

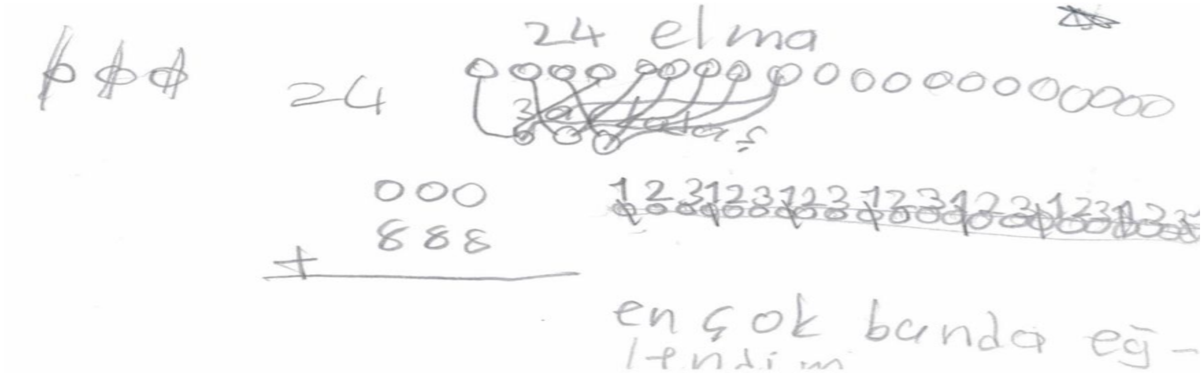


Figure 4

The strategy developed by the student made this problem the most enjoyable one for her throughout the sessions. Enjoyment is perhaps one of the most critical affective components in mathematics education. The sense of joy the student experienced while solving a challenging problem using her own creative approach highlights the affective dimension of self-invented strategies in mathematics, prompting further reflection on their emotional impact.

In another problem, "Aslı pays 45 TL for nine apples. How much does one apple cost?" the student instinctively said, "*Let's divide 45,"* and used visual representations (Fig. 5) to divide the total amount into five groups of nine, correctly concluding that each apple costs 5 TL. Her intuitive understanding of division as the inverse of multiplication demonstrated her ability to connect mathematical operations without formal instruction. This spontaneous decision to divide 45 was telling, especially when she stated, "*I didn't learn it from anywhere; I just know it*." This insight suggests that, even before formal education, students often have informal, intuitive ideas about the basic arithmetic operations. Notably, she had already developed an understanding of multiplication concepts even prior to formal instruction, which may contribute to her capacity for division.


Figure 5

Her reasoning in this instance was further reinforced by her ability to check her solution. Upon reviewing her work, she recognized she had counted by fives and concluded that 9 times 5 equals 45 and that each apple costs 5 TL. She then wrote the equation 9 × 5, reflecting her internalization of the division-multiplication relationship. This example highlights the student's ability to connect operations, demonstrating how her intuitive understanding of multiplication supported her reasoning in division. The student mentally transformed the problem into "*How many nines make 45?*"—a critical cognitive shift. This transformation, which is often missing even in children with formal instruction, is a remarkable finding, as it shows the student's capacity to discover a key mathematical concept independently.

However, the student occasionally faced challenges with more complex division problems, particularly those involving larger numbers or non-integer quotients. For example, when asked how many 6 TL bananas could be purchased with 42 TL, she initially struggled to develop a strategy. Nevertheless, she eventually counted by sixes, arriving at the correct answer (7). This self-invented counting strategy highlights her adaptability in solving division problems, even when her approach was not the most efficient. Interestingly, the student did not connect this problem to a similar transformation she had previously made, suggesting that her reasoning was situation-specific rather than generalized across contexts. These findings underscore the student's use of intuitive strategies, such as equal sharing and grouping, in solving division problems. While these strategies were helpful in basic division, her difficulties with larger numbers suggest that targeted instruction is necessary to help her transition to more efficient and formal methods. This highlights the importance of understanding students' intuitive strategies and providing them with structured guidance to refine and improve their mathematical reasoning.

## Creativity and adaptability in problem-solving

One of the most striking aspects of the student's problem-solving process was her creativity and adaptability. She frequently modified her strategies based on the problem at hand, demonstrating a flexible approach to mathematical reasoning. For example, when solving the problem, "Aslı has 7 bags of apples, and each bag contains 2 apples. How many apples does Aslı have in total?" she initially drew seven bags, each containing two apples (Fig. 6). However, she soon recognized that the problem could be represented as 7 × 2. This shift from concrete representations to abstract notation illustrates her ability to adapt her strategies as her understanding deepened.

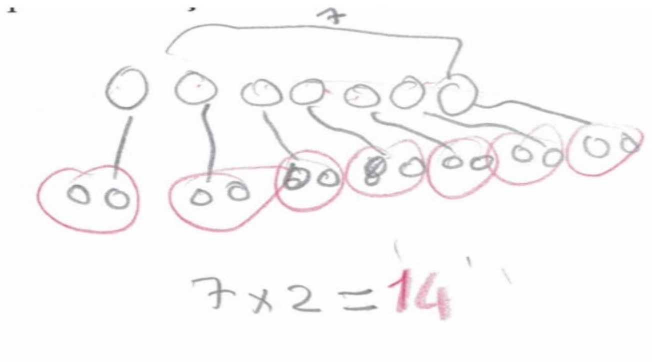


Figure 6

In another problem, "Aslı has 4 bags of apples, each containing 6 apples. How many apples does Aslı have in total?" the student started by drawing and arranging the bags, but what was particularly notable was her use of the correct mathematical expression, 4 × 6, to represent the multiplication (Fig. 7).

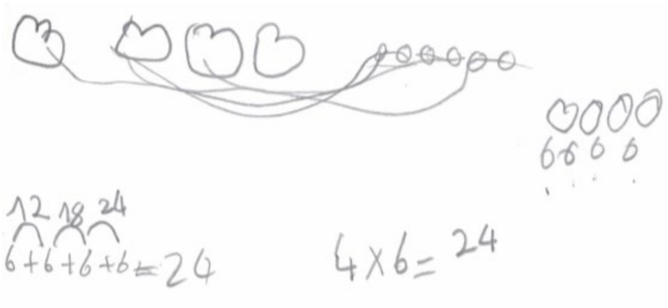


Figure 7

In this problem, the student connected each bag to six apples using lines rather than drawing all the apples, exemplifying a shift from pictorial to schematic representation. This transition suggests a reduced reliance on visual representation and a greater reliance on mathematical reasoning. This demonstrated her growing understanding of the multiplication concept and her capacity to move from a physical, visual approach to symbolic, abstract notation. This transition is significant as it shows how the student's problem-solving strategies evolved over time, reflecting her ability to adapt and apply different approaches as needed. These examples illustrate the student's creative problem-solving abilities, particularly her capacity to use multiple strategies flexibly, from concrete representations to more abstract symbolic expressions. Her adaptability in choosing the most appropriate strategy based on the problem at hand highlights her growing mathematical reasoning and deepening understanding of multiplication.

## Challenges

The student demonstrated significant progress in solving multiplication and division problems but also encountered various challenges throughout the problem-solving process. These difficulties, ranging from conceptual misunderstandings to the application of advanced strategies, highlighted areas where her reasoning required further development. The following examples illustrate these challenges and provide insights into the complexity of her mathematical reasoning.

When presented with the problem, "If one apple costs 5 TL, how much would Aslı pay for 7 apples?" the student immediately responded with, "*Ah, this is very easy*!" She restated, "*One apple is 5 TL*," and then summarized the problem as "*7 times 5,*" counting by fives to reach 35. When asked to explain her reasoning, she simply said, "*I counted 7 times 5*." It was exciting to see the student use the multiplication structure, “a times b,” on her own without formal instruction and explain it. I wondered whether she might have learned something about multiplication outside of school. When I asked her if she knew what multiplication meant, she responded, "*Yes, but it’s hard to explain*." She then wrote "5 times 5 (5x5)" to show how multiplication is written, saying she learned this from her sister. When I inquired further, she explained that her sister taught her a multiplication game that involved the multiplication table. Her family confirmed that the game helped her memorize specific multiplication facts, like "5x5=25," but she didn't fully understand how to apply these facts in problem-solving. Nevertheless, she used terms like "times" and "count," such as in "4 times 5" and "4 count of 5," to express multiplication correctly. This reinforced the fact that students don't come to school with empty minds and that prior formal or informal learning shapes their mathematical understanding.

When solving the problem, "If one apple costs 3 TL, how much would Aslı pay for 8 apples?" the student initially said, "*This is an addition problem*," and wrote 3 + 8. Just as she was about to finalize her answer, she decided to draw a representation, creating 8 apples and labeling each with 3 TL. She arrived at the correct total of 24 TL and wrote the expression 3 × 8. This example showed how her intuition guided her, allowing her to correct her initial misinterpretation (addition instead of multiplication) through visual representation (Fig. 8). She also illustrated each apple, writing "3" on each and counting by threes. However, it became apparent that she was confusing the concepts of "3 times 8" and "8 times 3," which might have contributed to some of her struggles with multiplication.

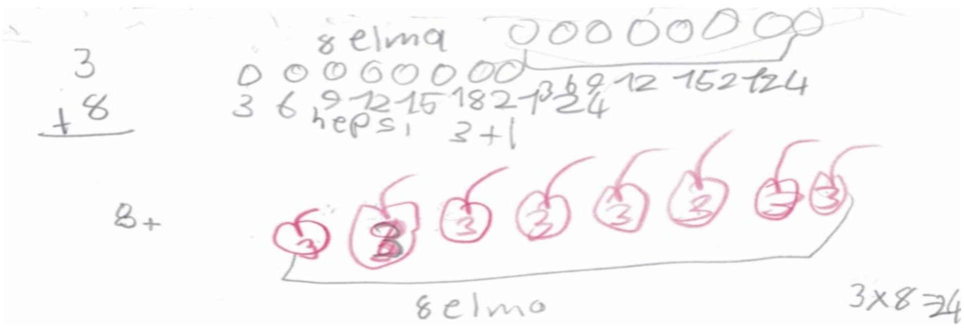


**Figure 8**

The problem "Selin bought bananas, each costing 6 TL, and paid 42 TL in total. How many bananas did she buy?" presented a different challenge. Unlike previous problems, the student did not immediately choose a strategy for this problem. After reading the problem several times, she began drawing, separating 42 into tens and ones to avoid representing 42 bananas. She created a shortcut of her own to divide the number into smaller parts, which was an insightful approach (Fig. 9). However, she struggled to develop an effective strategy to solve the problem. Reflecting on a similar problem she had solved intuitively earlier, it seemed this particular problem caused confusion due to its different structure.

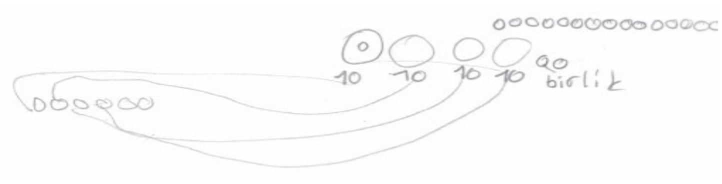


Figure 9

While the student showed strong problem-solving abilities in multiplication and division, certain challenges emerged, particularly when dealing with more abstract concepts or when she had to apply new strategies. For instance, she occasionally mixed up the order of factors in multiplication, leading to errors in reasoning. Additionally, she faced difficulties with division problems that required more advanced strategies, such as those involving remainders or larger numbers.

## Discussion

This study provides insights into how young learners naturally develop mathematical understanding, particularly in multiplication and division. The student's ability to construct self-invented strategies underscores the importance of fostering creativity and exploration in early math education. The findings demonstrate that children can progress from concrete representations to more abstract thinking—a developmental process well-supported by theories such as those of Vygotsky and Piaget. The student's use of tools, such as drawings and manipulatives, played a crucial role in facilitating this transition, reducing cognitive load while supporting her evolving mathematical understanding (Ergan & Özsoy, 2021). However, the observed struggles with the commutative property and division suggest critical stages in conceptual growth that require targeted instruction. The student's use of visual representations and grouping strategies also aligns with Vessonen et al.'s (2024) findings on the importance of visuospatial skills in mathematical problem-solving. Her reliance on drawings and manipulatives highlights how these tools supported her ability to construct and refine problem-solving strategies. The student's progression from repeated addition to recognizing patterns in multiplication aligns with McMillan's (2024) emphasis on grouping strategies as a foundation for understanding algebraic properties. For example, the student's ability to simplify calculations by grouping numbers demonstrates an emerging understanding of the distributive property, though she lacked explicit application of it. This finding resonates with Clerjuste et al. (2024), who highlight the importance of the distributive property in advancing students' understanding of multiplication. While effective, the student's reliance on intuitive grouping strategies indicates the need for structured interventions to bridge the gap between informal and formal mathematical reasoning.

In division, the student's intuitive strategies, such as equal sharing and grouping, reflect an emerging sense of proportional reasoning, as Vanluydt et al. (2024) noted. For instance, when dividing 24 apples among three friends, the student's equal-sharing strategy demonstrated her ability to conceptualize division as a process of fair distribution. However, her difficulties with larger numbers and more complex division problems suggest that targeted instruction is necessary to transition these intuitive strategies into more formal and transferable skills. Parviainen et al. (2023) emphasize that opportunities for developing early mathematical skills often depend on teachers' pedagogical awareness and professional development. Incorporating deliberate instructional practices that focus on mathematical thinking and reasoning alongside spatial and numerical skills could further support students, like the participants, in transitioning from intuitive strategies to formalized reasoning. These findings underline the importance of equipping educators with the necessary training and awareness to foster these skills consistently across different age groups.

## Conclusion

This study explored how a second-grade student, developed self-invented strategies to solve multiplication and division problems. The findings revealed a natural progression from concrete methods, such as repeated addition and visual representations, to more abstract reasoning, including pattern recognition and intuitive grouping. The student's creativity and adaptability were evident as she transitioned from hands-on tools to symbolic notation, demonstrating a deepening understanding of mathematical concepts. However, challenges with the commutative property and complex division tasks highlighted the need for targeted instruction to bridge gaps in understanding. These insights underscore the importance of fostering exploratory learning environments that build on students' intuitive strategies while providing structured support to refine their mathematical reasoning.

The implications of this study are significant for both practice and research in early mathematics education. The findings emphasize the value of recognizing and nurturing students' intuitive problem-solving approaches, especially in the absence of formal instruction. Educators should create learning environments that encourage students to develop and test their own strategies, supporting them with appropriate scaffolding as they move toward more abstract reasoning. Additionally, the study highlights the importance of identifying and addressing specific challenges, such as misconceptions related to the commutative property and division strategies, to ensure that students' foundational understanding is secure. By focusing on the development of self-invented strategies, this study suggests that a more personalized and flexible approach to mathematics education could help students build a deeper and more lasting understanding of mathematical concepts.

While this study provides insights into the development of self-invented strategies in early mathematics learners, there are important directions for future research. The focus on a single participant limits the generalizability of the findings, and it is crucial to explore whether similar patterns emerge in larger, more diverse samples. Future studies should examine a broader range of problem types to capture the full complexity of students' problem-solving abilities. Furthermore, extended observation periods would allow for a more comprehensive understanding of the evolution of students' strategies over time. To enrich our knowledge, future research could investigate how factors such as classroom environment, teacher interventions, and peer interactions influence the development of self-invented strategies. Additionally, longitudinal studies examining the long-term impact of such strategies on mathematical achievement and problem-solving confidence would be valuable. By expanding the scope and depth of future studies, researchers can build on the findings of this study and continue to advance our understanding of how young learners develop and refine their problem-solving skills.

## Statement of researchers

### Conflict statement

The author declares no conflicts of interest related to this study.

## References


Boaler, J. (2024). *Math-ish: Finding creativity, diversity, and meaning in mathematics*. HarperOne.

Clements, D. H., & Sarama, J. (2020). *Learning and teaching early math: The learning trajectories approach* (3rd ed.). Routledge. https://doi.org/10.4324/9781003083528

Clerjuste, S. N., Guang, C., Miller-Cotto, D., & McNeil, N. M. (2024). Unpacking the challenges and predictors of elementary-middle school students' use of the distributive property. *Journal of Experimental Child Psychology, 244*, 105922. https://doi.org/10.1016/j.jecp.2024.105922

Cyr, J., & Goodman, S. W. (2024). *Doing good qualitative research*. Oxford University Press.

Ergan, S. N., & Özsoy, G. (2021). İlkokul dördüncü sınıf öğrencilerinin problem çözme sürecinde oluşturduğu görsel temsillerin incelenmesi [Examination of the visual representations created by fourth-grade students in the problem-solving process]. *Dokuz Eylül Üniversitesi Buca Eğitim Fakültesi Dergisi, 51*, 57-75. https://doi.org/10.53444/DEUBEFD.763452

Haylock, D., & Manning, R. (2019). *Mathematics explained for primary teachers* (6th ed.). SAGE.

Hulbert, E. T., Petit, M. M., Ebby, C. B., Cunningham, E. P., & Laird, R. E. (2023). *A focus on multiplication and division: Bringing mathematics education research to the classroom* (2nd ed.). Routledge. https://doi.org/10.4324/9781003185529

Jonsson, B., Mossegård, J., Lithner, J., & Karlsson Wirebring, L. (2022). Creative mathematical reasoning: Does need for cognition matter? *Frontiers in Psychology, 12*. https://doi.org/10.3389/fpsyg.2021.797807

McMillan, B. G. (2024). Connecting student development of use of grouping and mathematical properties. *The Journal of Mathematical Behavior, 74*, 101154. https://doi.org/10.1016/j.jmathb.2024.101154

Parviainen, P., Eklund, K., Koivula, M., Liinamaa, T., & Rutanen, N. (2023). Teaching early mathematical skills to 3- to 7-year-old children - Differences related to mathematical skill category, children's age group and teachers' characteristics. *International Journal of Science and Mathematics Education, 21*(7), 1961-1983. https://doi.org/10.1007/s10763-022-10341-y

van Oers, B. (2024). The development of mathematical thinking in young children's play: The role of communicative tools. In H. Palmér, C. Björklund, E. Reikerås, & J. Elofsson (Eds.), *Teaching mathematics as to be meaningful - foregrounding play and children's perspectives: Results from the POEM5 Conference, 2022* (pp. 1-12). Springer International Publishing. https://doi.org/10.1007/978-3-031-37663-4_1

Vanluydt, E., De Keyser, L., Verschaffel, L., & Van Dooren, W. (2024). Stimulating early proportional reasoning: An intervention study in second graders. *European Journal of Psychology of Education, 39*(2), 607-628. https://doi.org/10.1007/s10212-023-00696-3

Vessonen, T., Hellstrand, H., Aunio, P., & Laine, A. (2024). Individual differences in mathematical problem-solving skills among 3- to 5-year-old preschoolers. *International Journal of Early Childhood, 56*(2), 339-357. https://doi.org/10.1007/s13158-023-00361-2

Walle, J. A. V. D., Karp, K. S., & Bay-Williams, J. M. (2016). *Elementary and middle school mathematics: Teaching with a developmental approach*. Nobel Academic Publishing.

Zager, T. J. (2023). *Becoming the math teacher you wish you'd had: Ideas and strategies from vibrant classrooms*. Routledge.